

Low frequency content of Airguns

Inherent analog low-cut limiting the low frequency output of Airguns (and seismic sources in general)

Physics of Air Guns and Sources in general

1. **Radiating oscillators (Rayleigh's equation)**

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p - p_\infty}{\rho_\infty} + \frac{R}{\rho_\infty c_\infty}\dot{p} - \alpha\dot{R}$$

Rayleigh-Willis formula

$$T = \frac{k\,(PV)^{1/3}}{(10+\text{Depth})^{5/6}}$$

1. **Far field. Applies to sources in general (not only air guns but also vibes)**

$$\Delta p(r,t) = \rho_\infty \frac{\ddot{V}(t - r/c_\infty)}{4\pi r}$$

R = bubble radius; V= Bubble volume;
p = pressure; r = distance to receiver; ∞ = water

3. **Ideal Gas equation**

   PV = nRT

4. **Adiabata equation**

   PV^γ = Constant

Ziolkowski, Hatton, Landro
Paul Chelminski
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Inertial terms | Pressure | Acoustic radiation | Damping

Rayleigh-Willis formula

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1. **Far field. Applies to sources in general (not only air guns but also vibes)**

2nd time derivative of displaced volume of water ~ frequency^2

Time delay

$$\Delta p(r,t) = \rho_\infty \frac{\ddot{V}(t - r/c_\infty)}{4\pi r}$$

Spherical spreading

R = bubble radius; V= Bubble volume;
p = pressure; r = distance to receiver;

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4. **Adiabata equation**

   PV^γ = Constant

   γ = 1.4

   Temperature changes

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Far field signature and bubble volume

- The far field pressure wave is proportional to the 2nd time derivative of the volume(time)

$$\Delta p(r,t) = \rho_\infty \frac{\ddot{V}(t - r/c_\infty)}{4\pi r}$$

This relation applies to any source including both airguns and vibes

Fourier Transform of 2nd derivative is frequency^2

Low cut!

$$\Delta p(r,t) = \rho_\infty \frac{\ddot{V}(t - r/c_\infty)}{4\pi r}$$

- Newton taught us that Force = Mass time acceleration
- d^2V/dt^2 is volumetric acceleration and ρ is volumetric mass
- So $\rho\, d^2V/dt^2$ is a good start
- But the dimension of $\rho\, d^2V/dt^2$ miss a *length* in the numerator

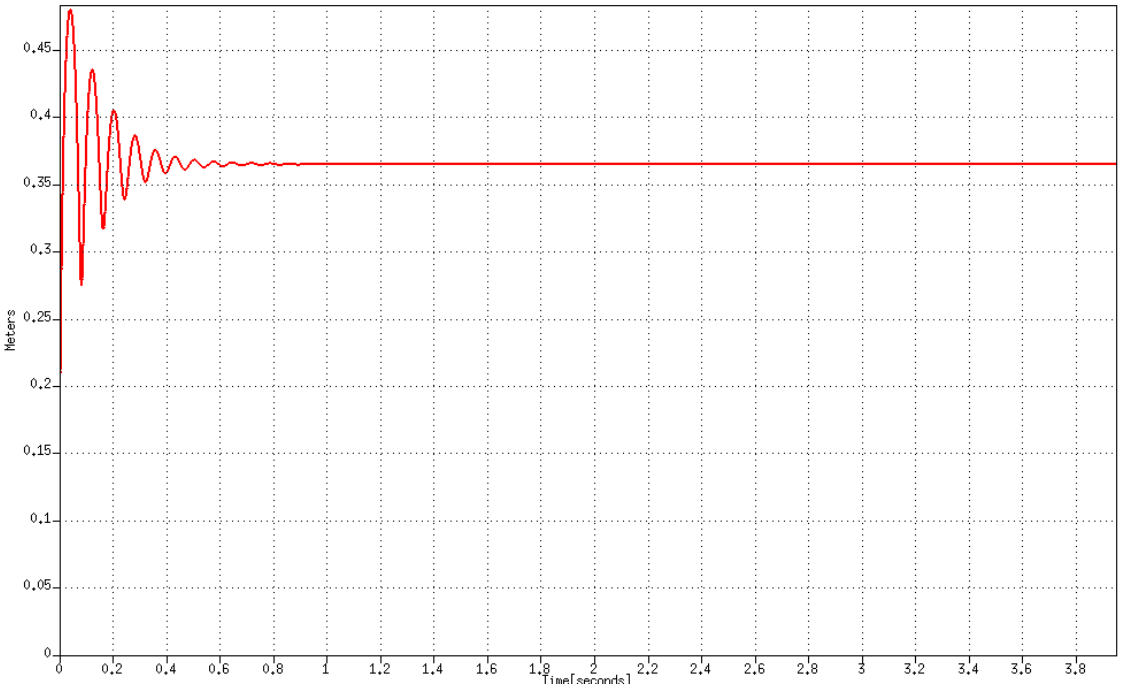
- That *length* "must be" the stroke of the bubble. Let's call it r
- So the force is $\rho\, d^2V/dt^2\, r$
- Pressure is force / area
- The area of a sphere is $4\pi r^2$
- So the pressure is $\rho\, d^2V/dt^2 Vr/4\pi r^2 = \rho d^2V/dt^2 V/4\pi r$
- This holds for any r, not only at the edge of the bubble

$$\Delta p(r,t) = \rho_\infty \frac{\ddot{V}(t - r/c_\infty)}{4\pi r}$$

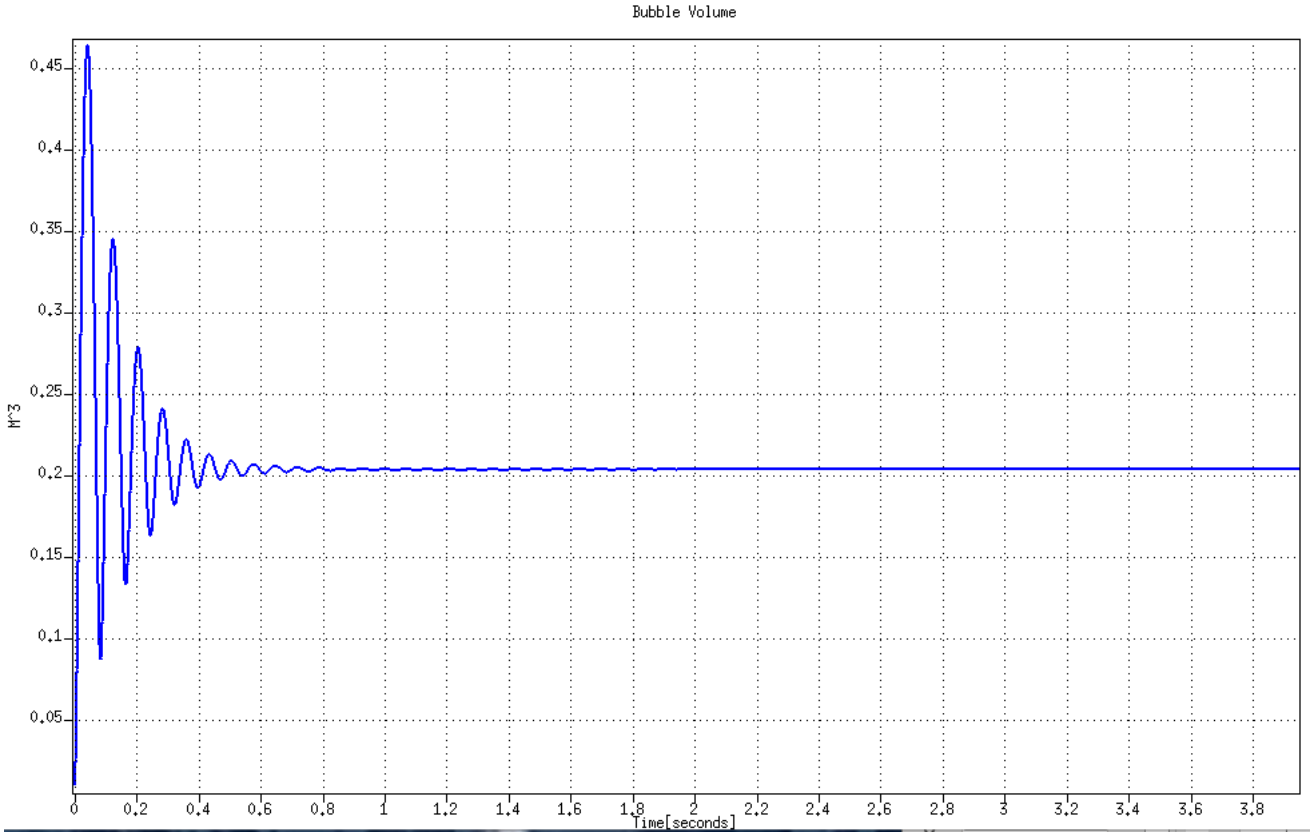
- The far field is proportional to the second time derivative of the volume of the water displaced by the source
- Second time derivative is multiplication by frequency square in the Fourier spectra domain
- Frequency square means 12 dB/octave low cut
- But the spectrum of the volume V may not be flat!

Oscillating bubble radius as a function of time

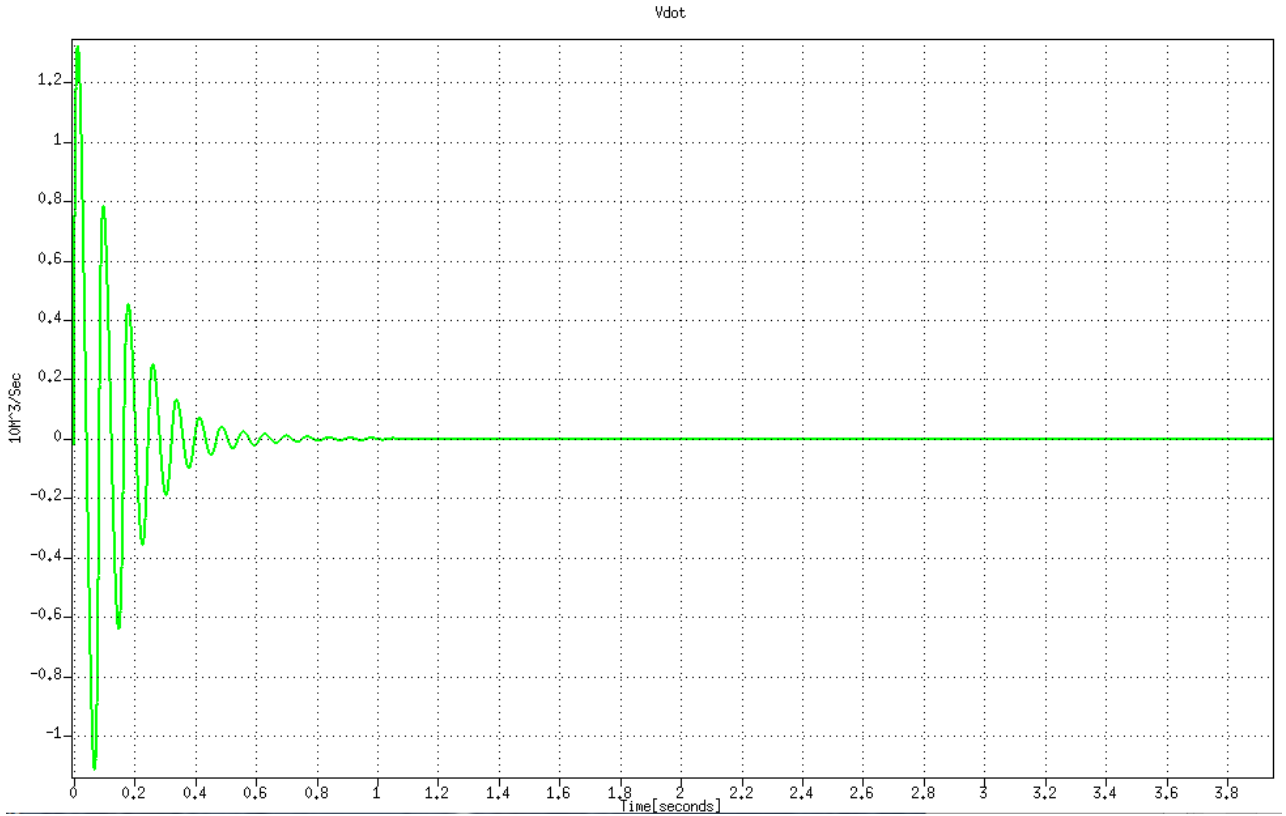
$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p - p_\infty}{\rho_\infty} + \frac{R}{\rho_\infty c_\infty}\dot{p} - \alpha\dot{R}$$



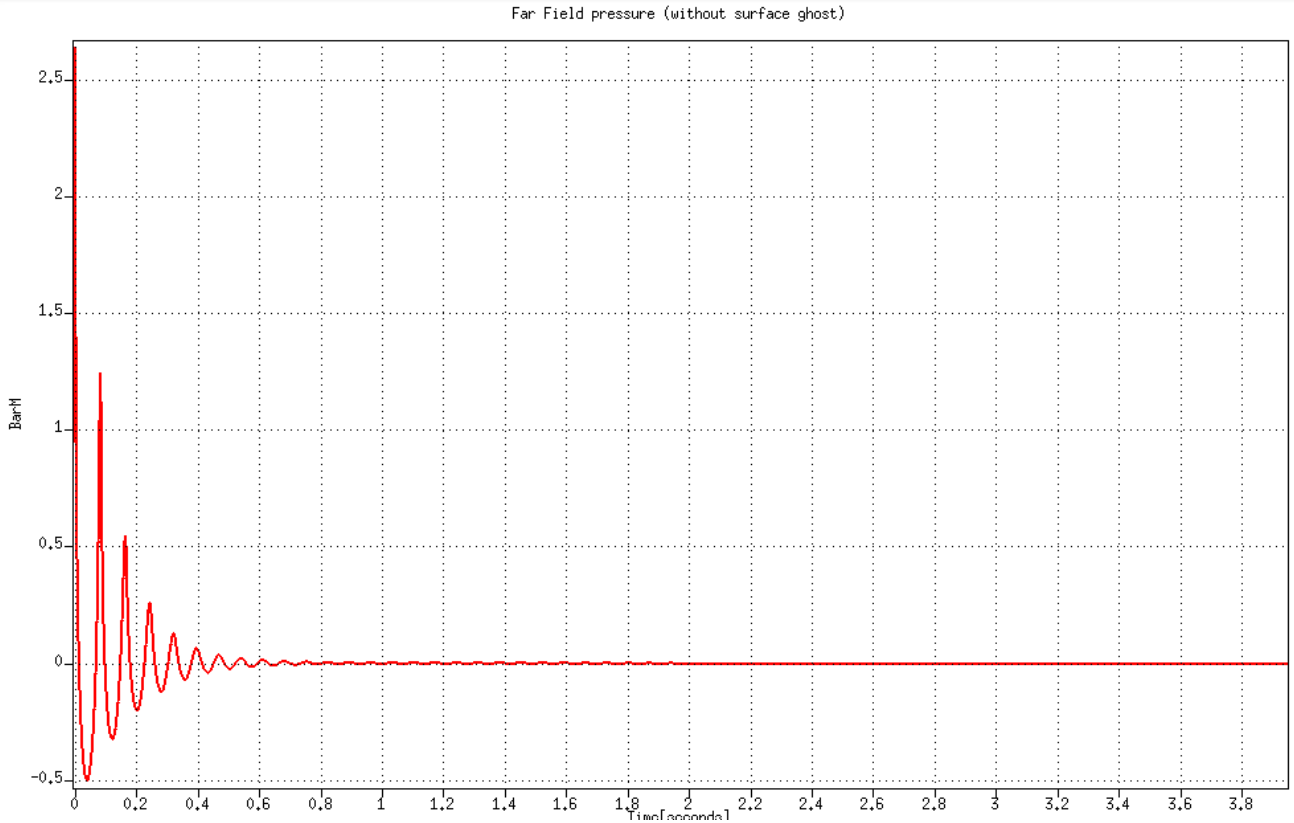
Bubble Volume = $4\pi r^3/3$



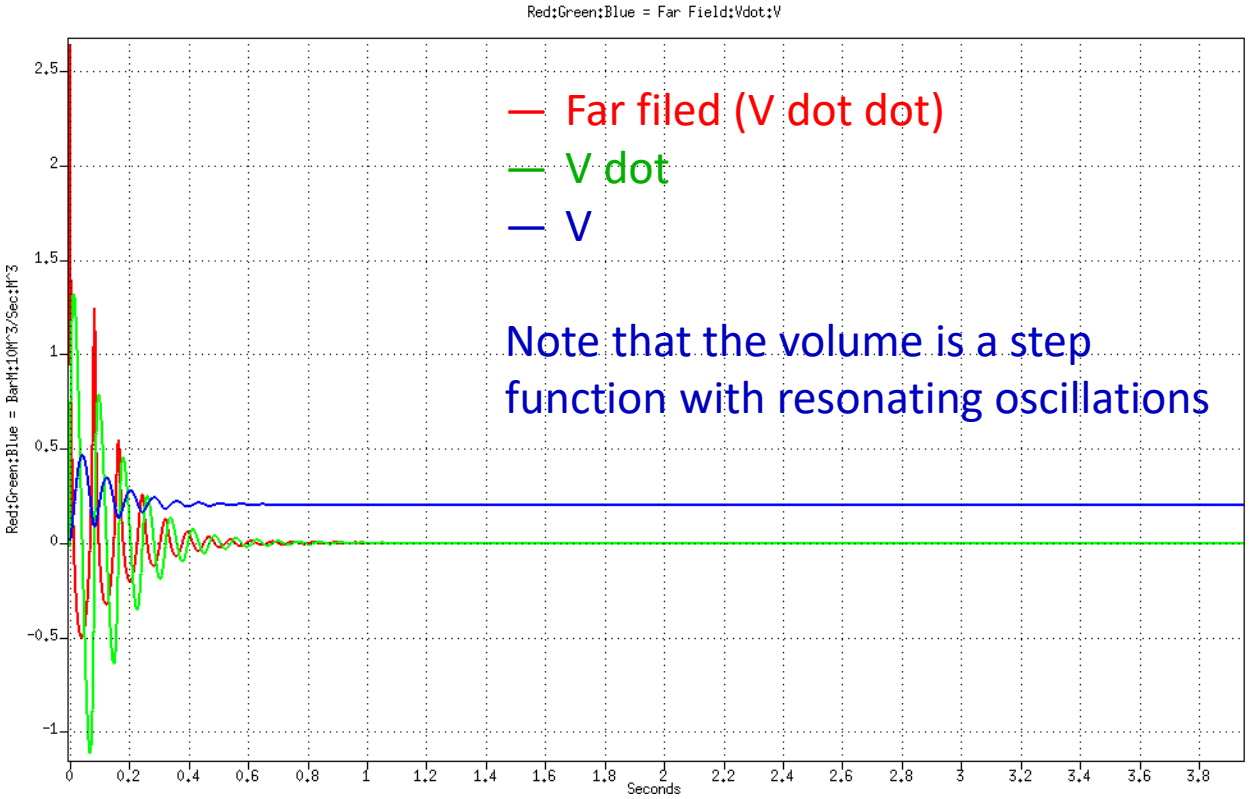
Volume velocity (V Dot)



Far field signature (proportional to V Dot Dot)

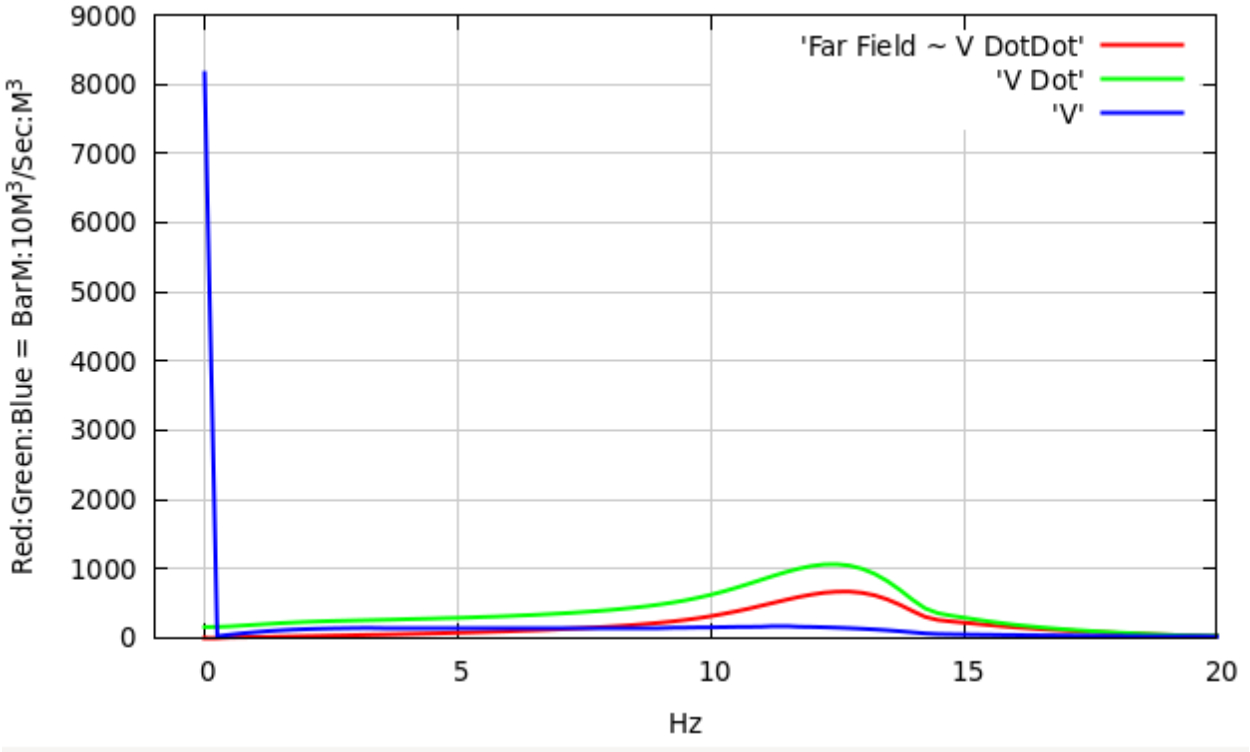


All three: Volume, its velocity, and acceleration



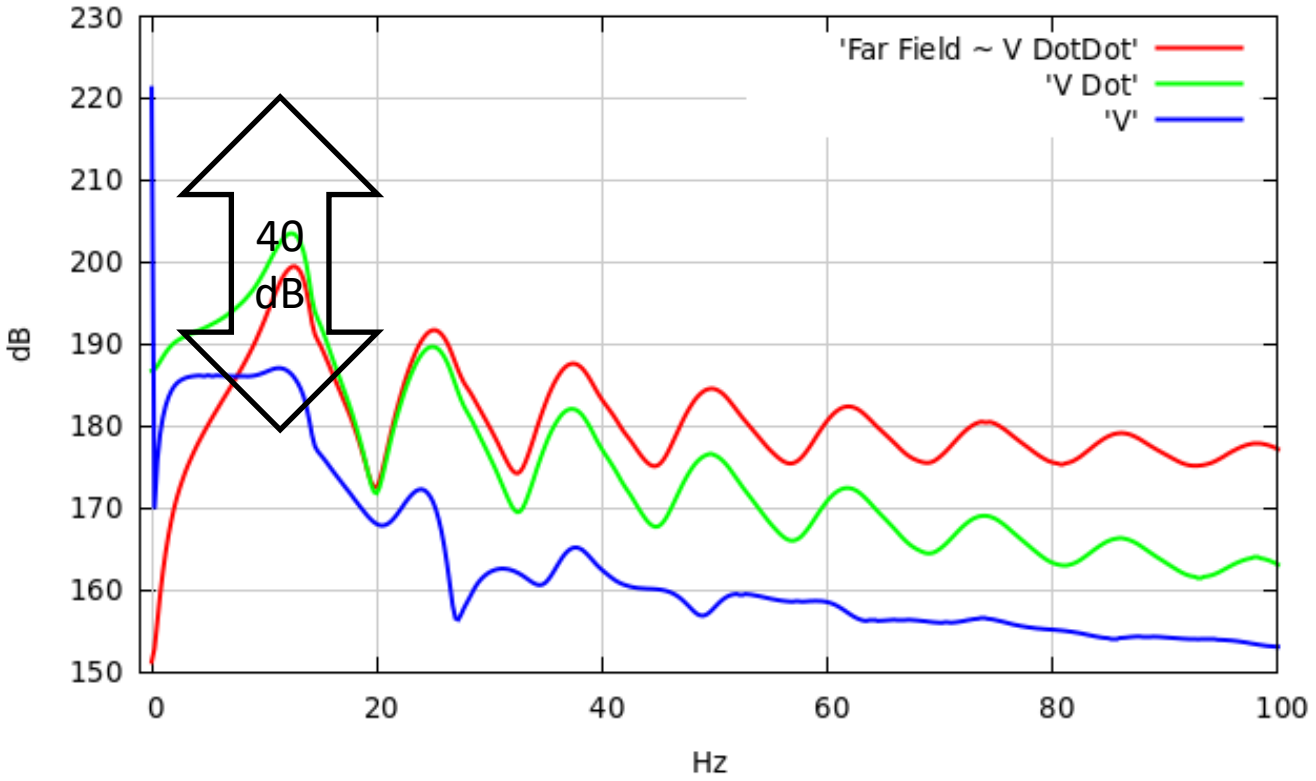
Spectra

- The volume (blue) has a pole at 0 frequency
- I plot the spectra as a function of Frequency, not log Frequency because the log of zero frequency is – infinity
- Next, I will plot the spectra against log frequency to see how linear it is at low frequencies



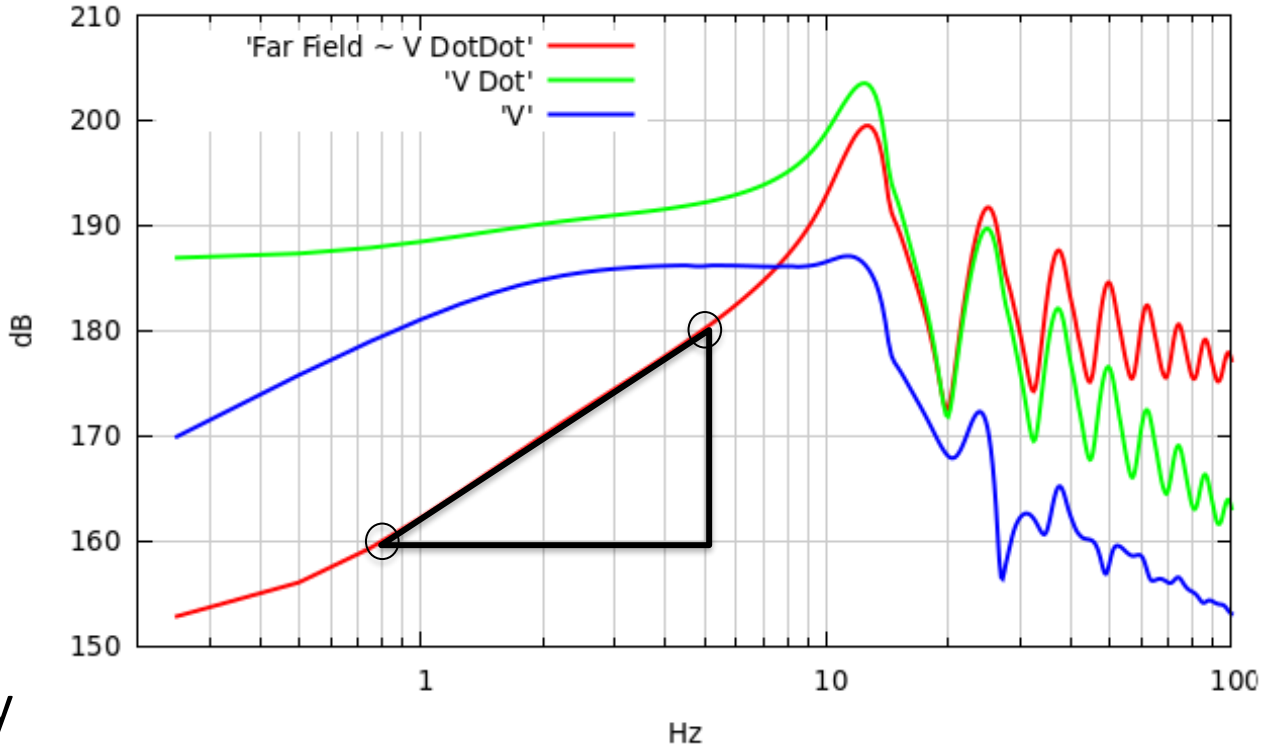
Spectra in dB

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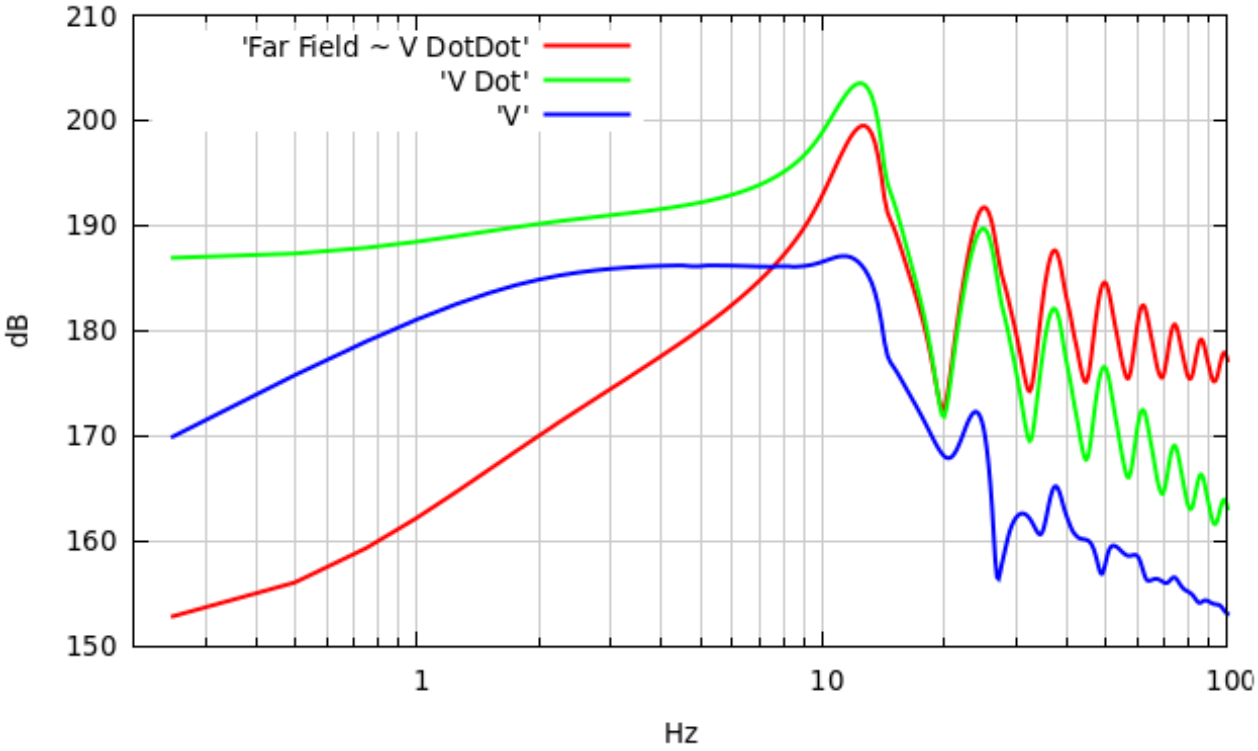
Spectra in log-log

- Straight line
  - 180 dB at 5 Hz
  - 160 dB at 0.8 Hz
- 180-160 = 20 dB
- 5/0.8 = 6.25 is 2.64 octaves
- 20/2.64 = 7.6 dB/octave
- The green has 1.6 dB/octave
- The blue is missing its pole at zero frequency



Spectra

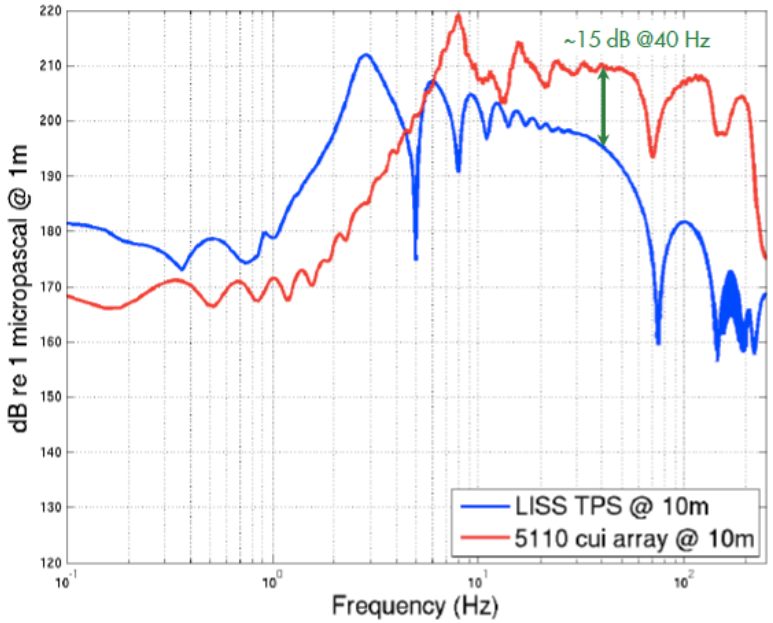
- The volume V is approximately a step function
- The volume velocity, V dot, is therefore approximately a spike so flat spectrum
- If V dot has a flat spectrum then V dot dot would have a low cut of 6 dB/octave
- It comes out, using numerical modeling, about 7.6 dB/octave



Summary

- The far field is proportional the the volumetric acceleration
- This is applicable to any seismic source
- Pneumatic sources (Airguns) release compressed air into the water
- The compressed air expands while oscillating
- Ignoring the fact that the bubble goes up to the surface, the volume is approximately a step function
- The time derivative of a step function is a spike
- A spike has a flat spectrum
- The spectrum of a time derivative is a low cut of 6 dB/octaves
- If V dot is flat then V dot dot would have 6 dB/octave
- V dot is not exactly flat
- Instead of 6 dB/octave we see 7.6 dB/octave

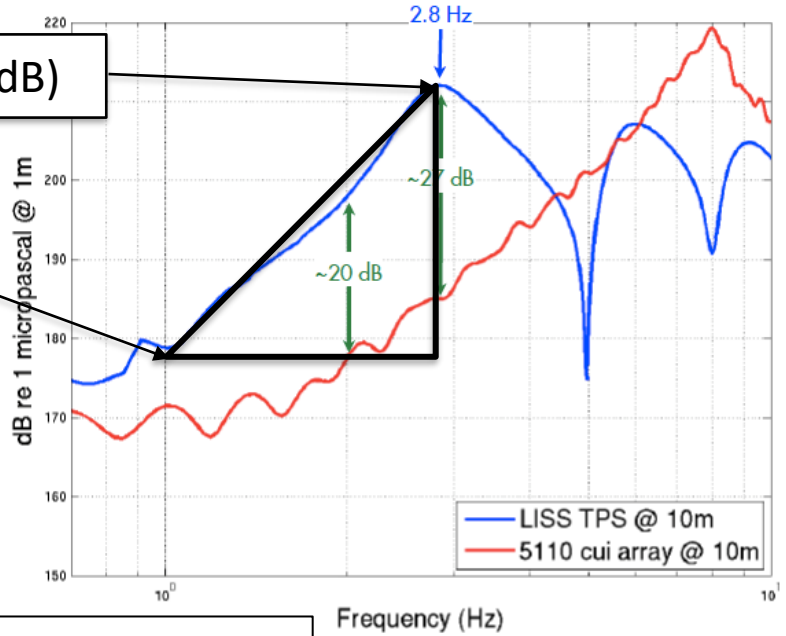
OBN data (8Hz Airgun and TPS)



OBN data (8Hz Airgun and TPS)

(2.8Hz, +212 dB)

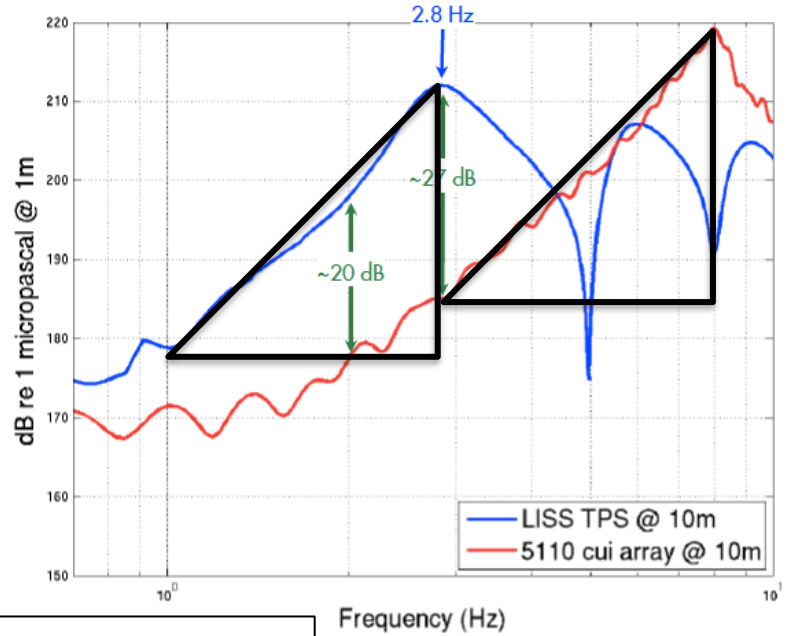
(1Hz, 178 dB)



2.8→1Hz = 1.5 octave
212-178 = 34 dB
34/1.5 = 23 dB/octave

Expecting 7.6+6 (the ghost) so less than 14 db/octave
We are seeing a difference of 9 dB/octave Data Vs theory!

OBN data (8Hz Airgun and TPS)

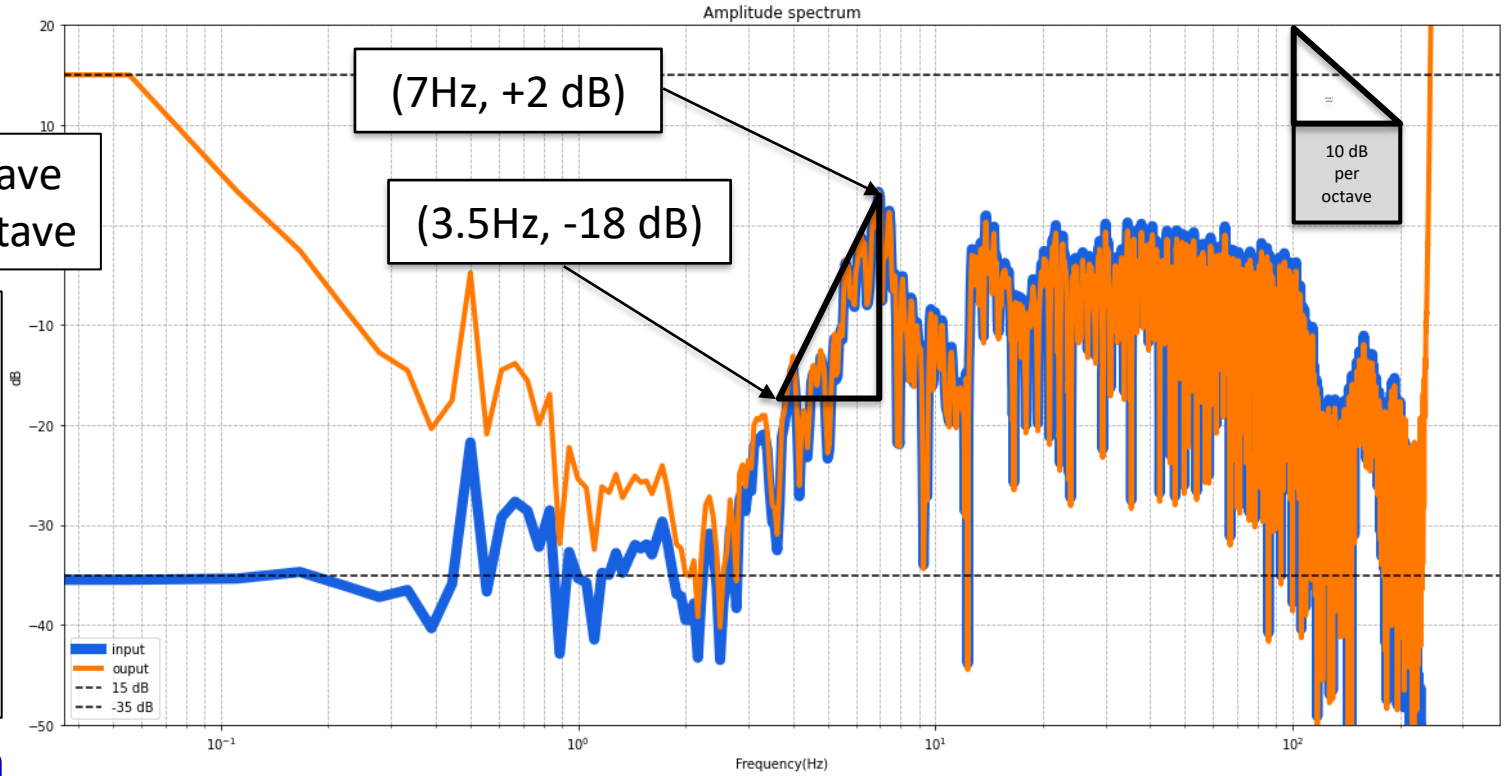


Expecting 7.6+6 (the ghost) so less than 14 db/octave
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OBN data—hydrophone (designatured)

7→3.5Hz = 1 octave
2--18 = 20 dB/octave

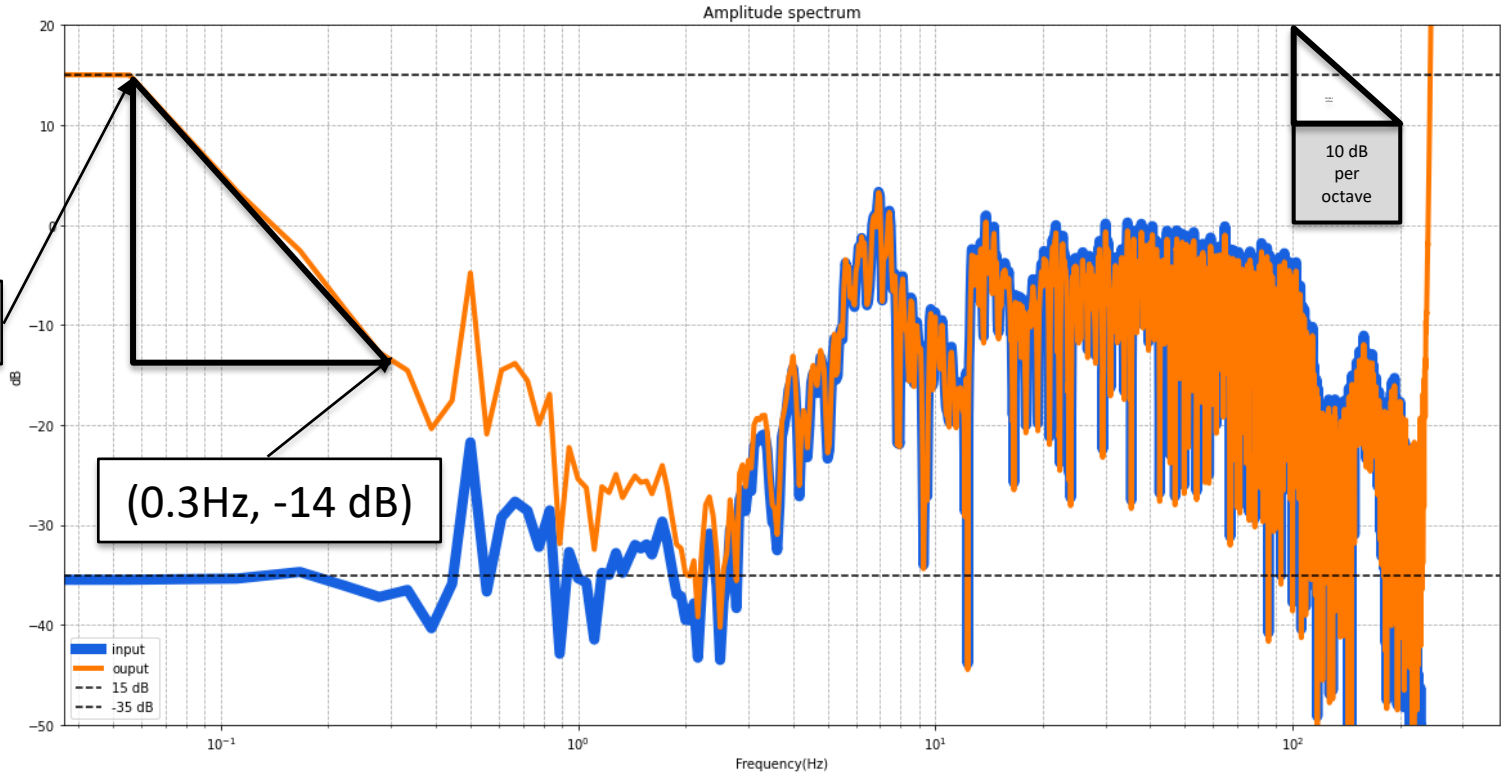
Expecting 7.6+6 (the ghost) so less than 14 db/octave We are seeing a difference of 6 dB/octave Data Vs theory!



— Raw data

— After analog RC lowcut designature

OBN data—hydrophone (designatured)



(0.05Hz, 15 dB)

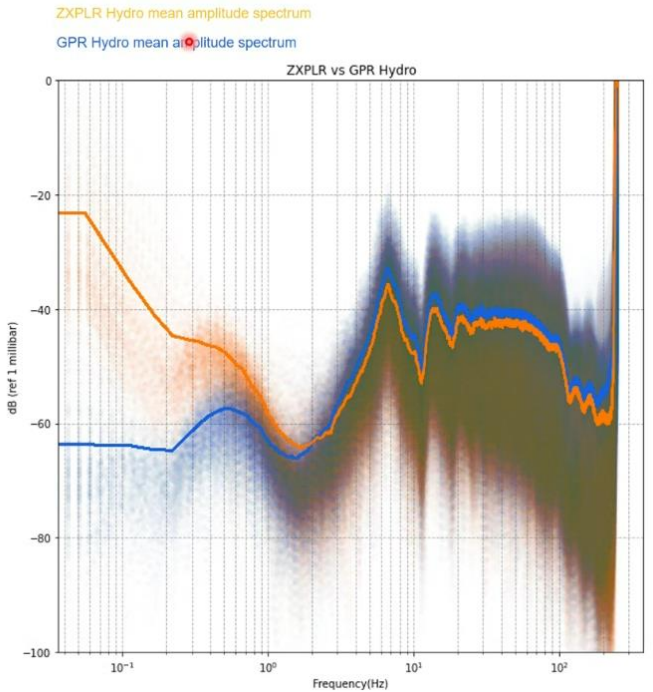
(0.3Hz, -14 dB)

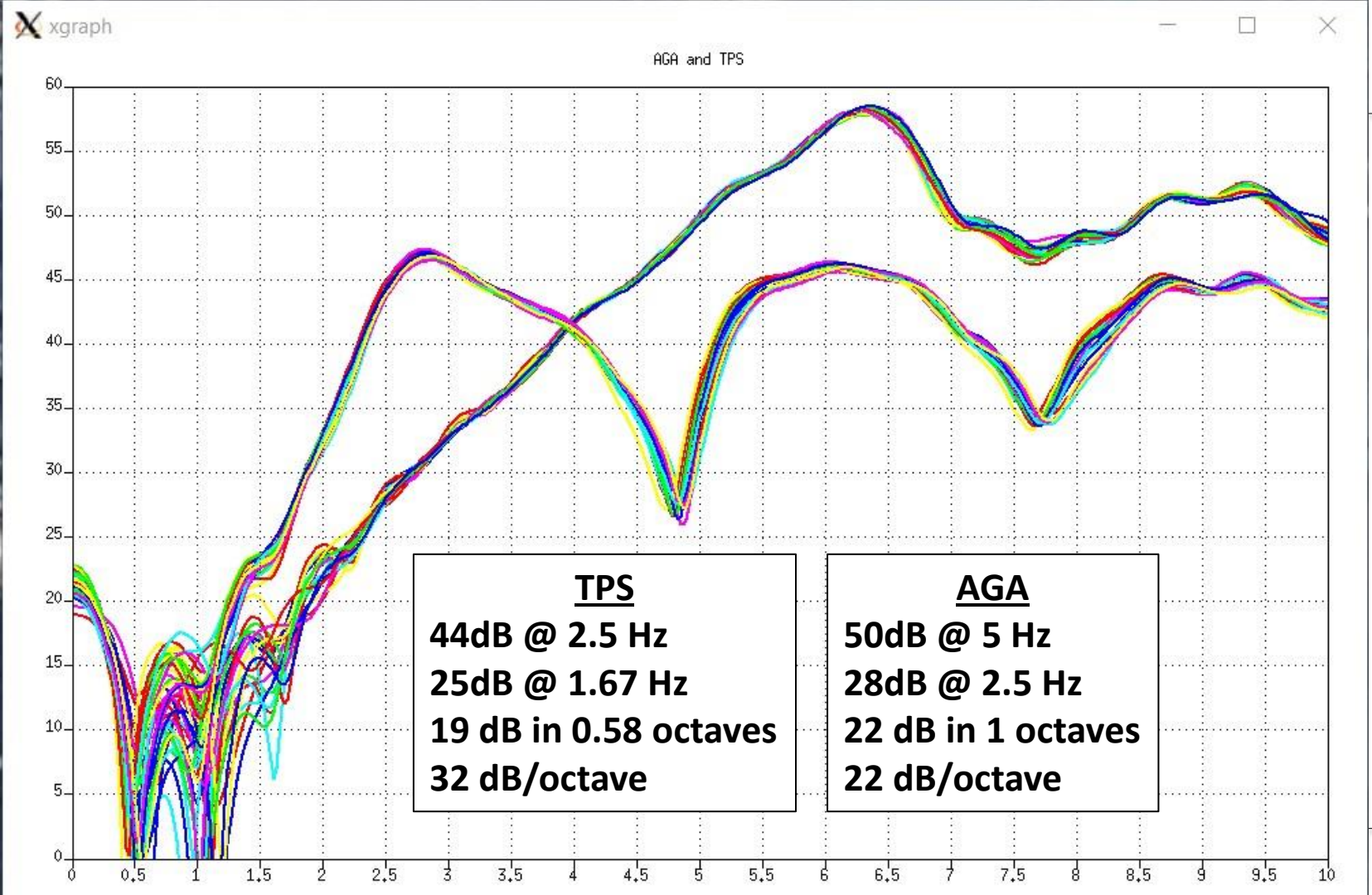
0.3→0.055Hz
= 2.45 octave
15--14 = 29 dB
29/2.45
= 12 dB/octave

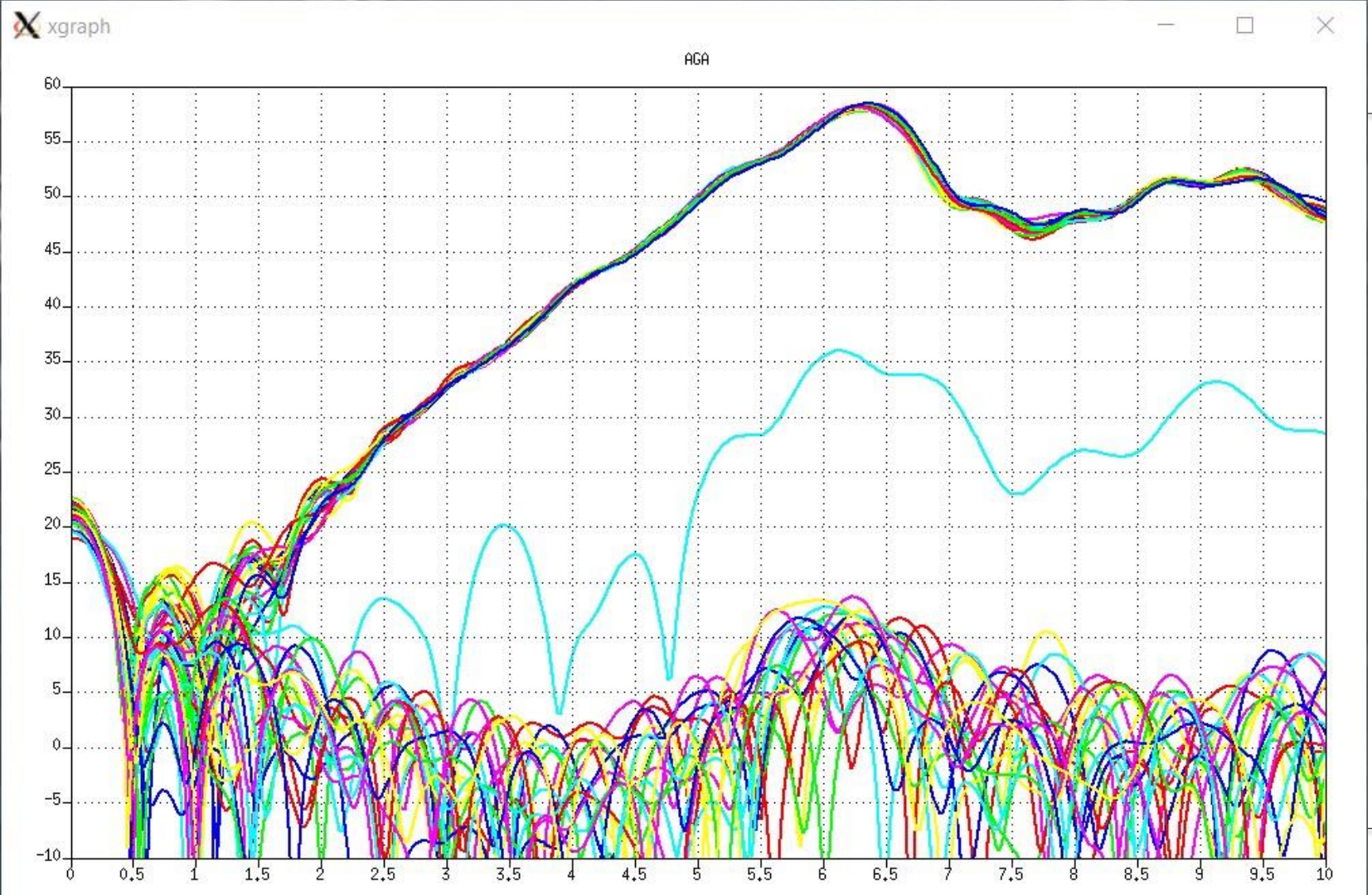
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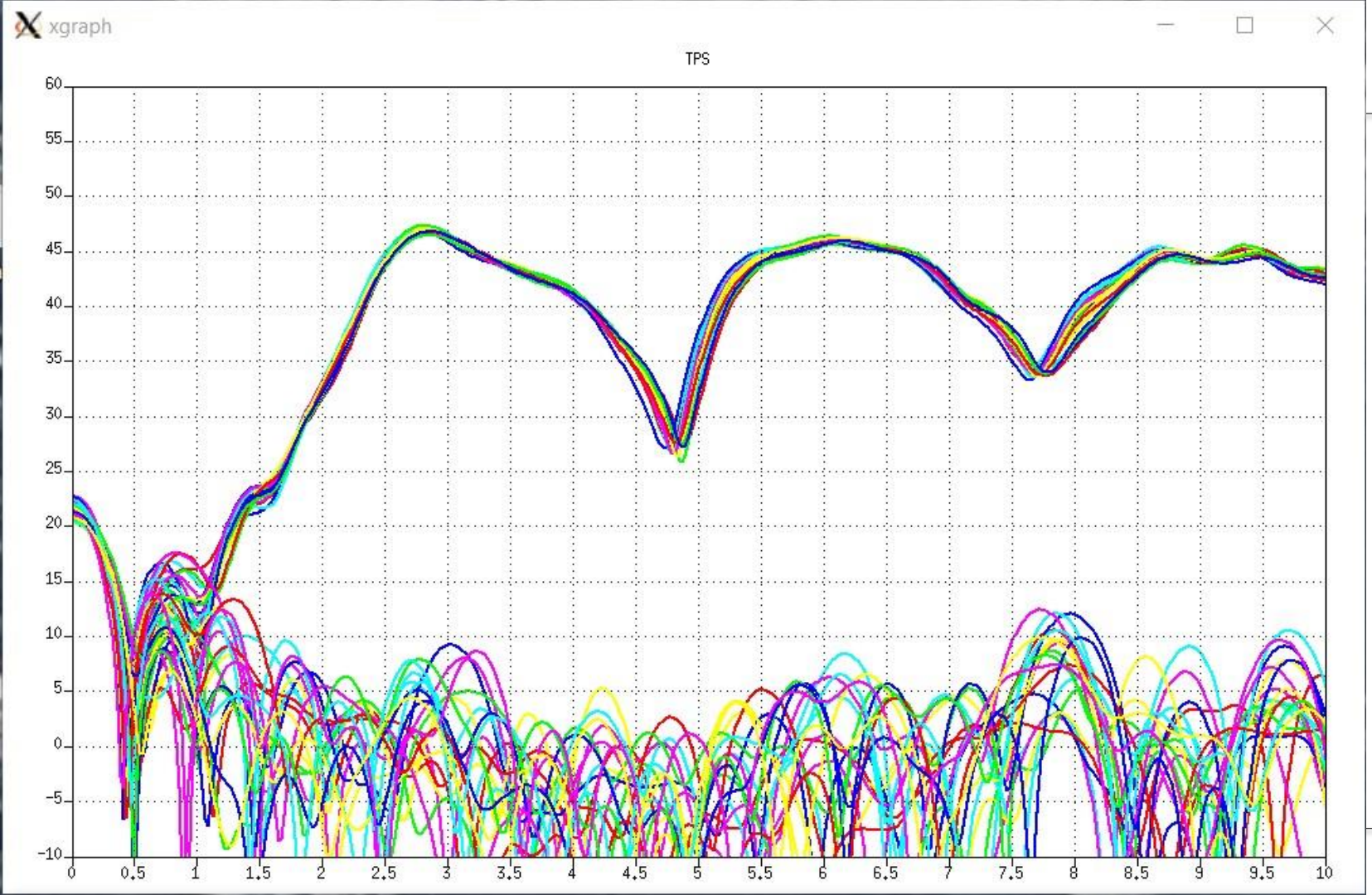
— After analog RC lowcut designature

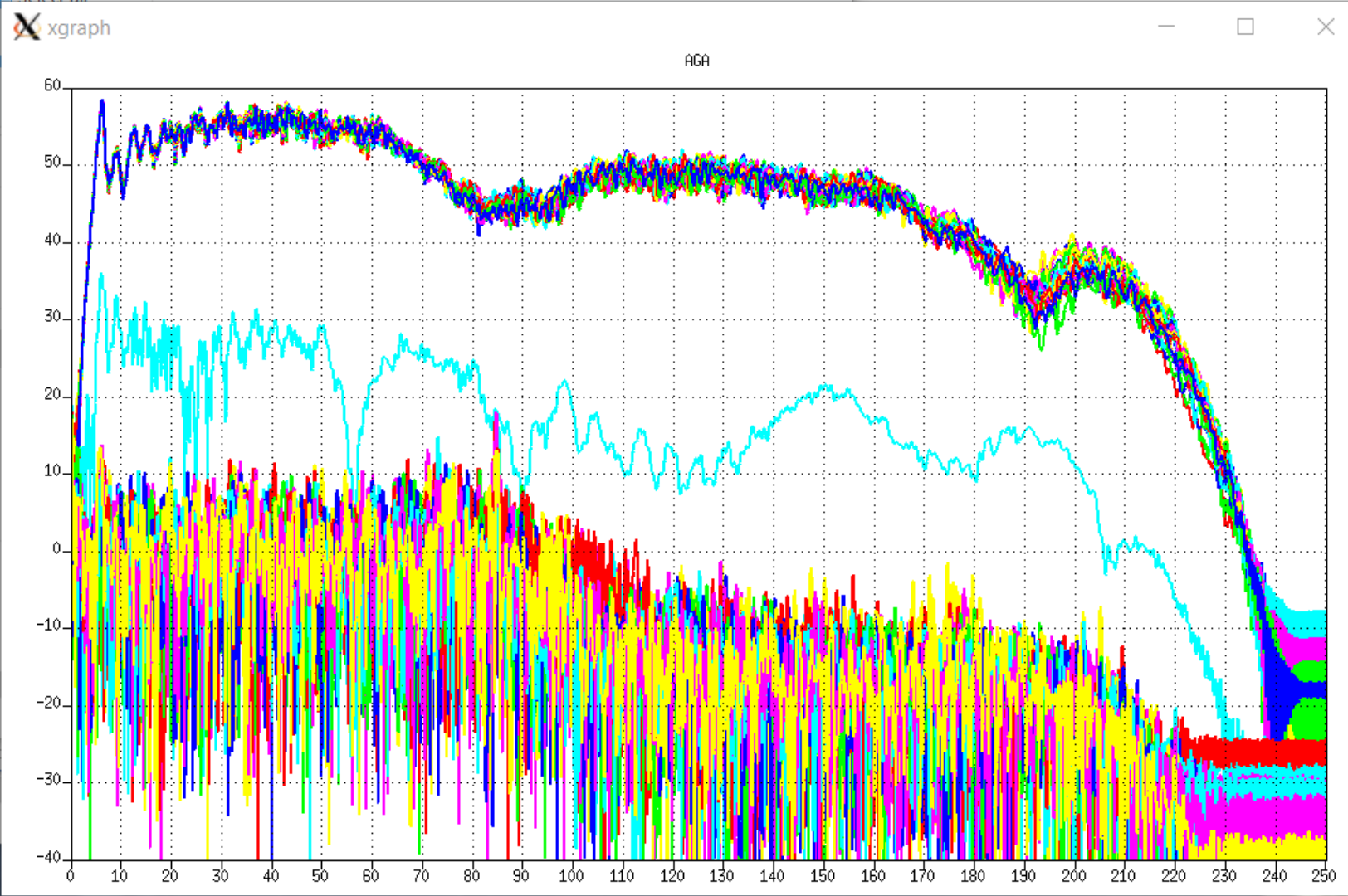
Hydrophone : GPR vs ZXPLR (400 shots)

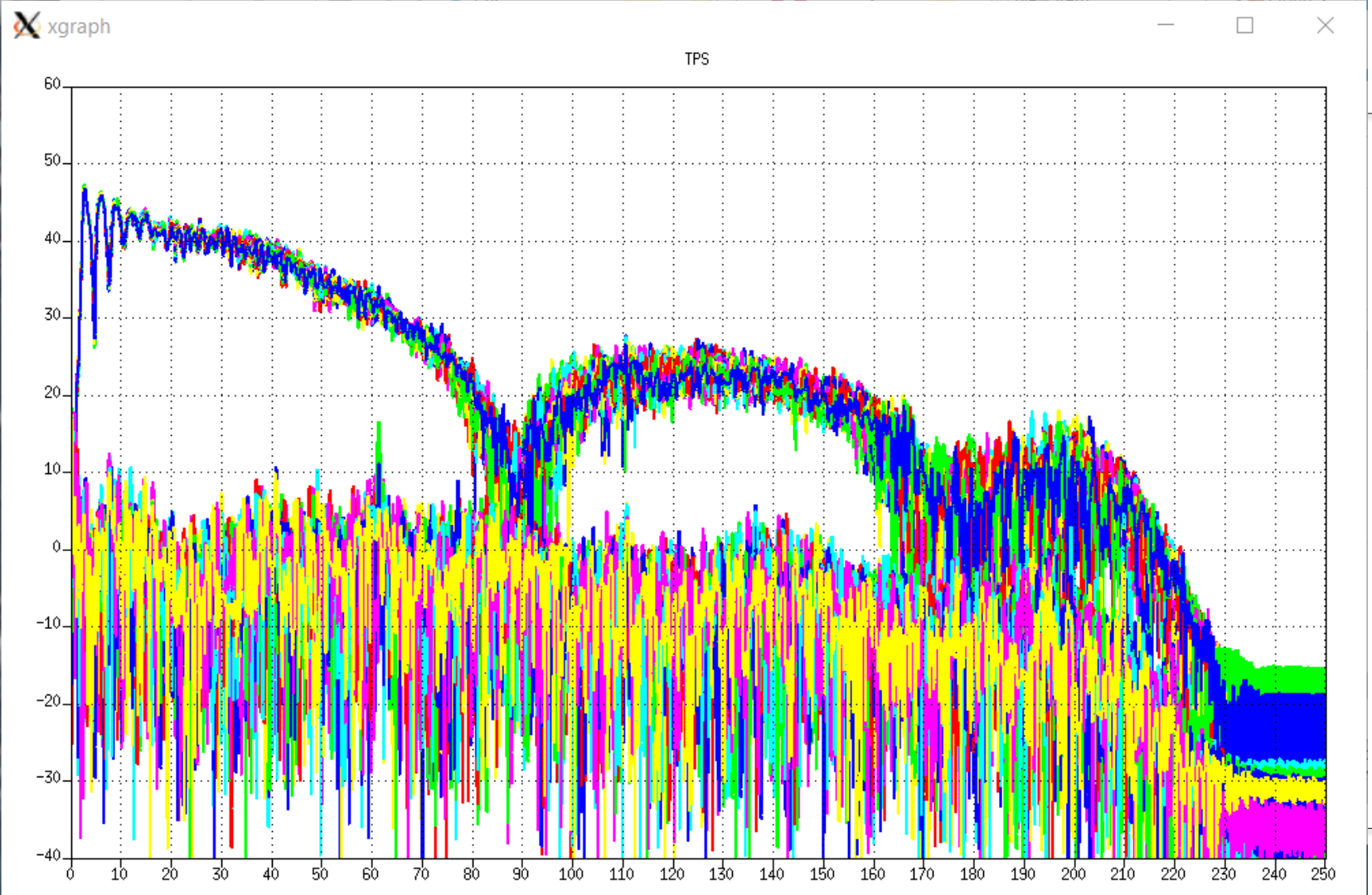



xgraph
AGA and TPS
60
55
50
45
40
35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
TPS
44dB @ 2.5 Hz
25dB @ 1.67 Hz
19 dB in 0.58 octaves
32 dB/octave
AGA
50dB @ 5 Hz
28dB @ 2.5 Hz
22 dB in 1 octaves
22 dB/octave


xgraph
AGA
60
55
50
45
40
35
30
25
20
15
10
5
0
-5
-10
0
0.5
1
1.5
2
2.5
3
3.5
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4.5
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5.5
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6.5
7
7.5
8
8.5
9
9.5
10


xgraph
TPS
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xgraph
AGA
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230
240
250


xgraph
TPS
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