(1) $\boxed{\rho\ddot{\underline{u}} = -\nabla P + \underline{S}}$ Newton ρ = density $u(r,t)$ = displacement

(2) $\boxed{P = -k\,\nabla\cdot\underline{u}}$ Hooke $P(r,t)$ = Pressure deviation

$S(r,t)$ = Source $\approx \delta(r)s(t)$

k = incompressibility

$\ddot{P} = -k\nabla\ddot{u}$

$\ddot{u} = \frac{1}{\rho}(\nabla P + S) \rightarrow \ddot{P} = k\cdot\nabla\frac{1}{\rho}(\nabla P - S) = \frac{k}{\rho}(\nabla^2 P - \nabla\cdot\underline{S})$

$c^2 = \frac{k}{\rho} \Rightarrow \boxed{\frac{1}{c^2}\ddot{P} = \nabla^2 P - \nabla\cdot\underline{S}}$ wave equation

$S \approx \delta(r)s(t)$ $S=0$ for $r > r_0 \ll \lambda \Rightarrow \frac{1}{c^2}\ddot{P} = \nabla^2 P$ $r>r_0$

Spherical symmetry: $\nabla^2 P = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rP)$ $\nabla P = \frac{\partial P}{\partial r}$

$\frac{1}{c^2}(r\ddot{P}) = \frac{\partial^2}{\partial r^2}(rP) \Rightarrow rP = g(t - r/c)$

(3) $\Rightarrow \boxed{P(r,t) = \frac{g(t-r/c)}{r}}$ for $r > r_0$

We want to connect $g(t)$ to $\nabla\cdot\underline{S}$ expecting $g(t) = \frac{\rho\ddot{V}(t)}{4\pi}$

V = Volume

$\ddot{V} = 4\pi r^2\cdot\ddot{u}$ so multiply both sides by $4\pi r^2$

(4) $\Rightarrow \rho\ddot{V} = \rho 4\pi r^2\ddot{u} = -4\pi r^2\frac{\partial P}{\partial r} + 4\pi r^2 S$ $\Leftarrow \nabla P = \frac{\partial P}{\partial r}$

(5) $r>r_0 \Rightarrow \rho\ddot{V} = -4\pi r^2\frac{\partial}{\partial r}\left(\frac{g(t-r/c)}{r}\right) = -4\pi r^2\left[\underbrace{\frac{-g(t-r/c)}{r^2}}_{\text{Near Field}} - \frac{1}{c}\underbrace{\frac{\dot{g}(t-r/c)}{r}}_{\text{Far Field}}\right]$

$r_0 \neq 0$ but it is small, in the near field

$\hookrightarrow \rho\ddot{V} = 4\pi g(t-r/c) = 4\pi r P(r,t)$

$\Rightarrow P = \frac{\rho\ddot{V}}{4\pi r}$ I just need the $\frac{r}{c}$ time delay!?

r_0 is far enough from $r=0$ for the Source to be zero but close enough to be (i) in the near field (2) $r_0/c \sim 0$

Eq (5) @ $r=r_0$ $\rho\ddot{V}(r_0,t) = -4\pi r_0^2\left[\frac{-g(t-r_0/c)}{r_0^2} - \frac{1}{c}\frac{\dot{g}(t-\frac{r_0}{c})}{r_0}\right]$

Near field $\Rightarrow r_0/c = 0$ $\frac{1}{r_0^2} >> \frac{1}{c}\frac{\text{freq}}{r_0} \simeq \frac{1}{\lambda r_0} \Leftrightarrow \lambda >> r_0$

$\Rightarrow$ (6) $\rho\ddot{V}(r_0,t) = -4\pi r_0^2\left[\frac{-g(t)}{r_0^2}\right] = +4\pi g(t)$

(7) Eq (3) w $\pm$: $P(r,t) = \frac{g(t \pm r/c)}{r} \Leftrightarrow g(t) = rP(r, t \mp \frac{r}{c})$

(6)&(7) $\Rightarrow \rho\ddot{V}(r_0,t) = 4\pi r P(r, t \mp \frac{r}{c})$ Near the source

Causality $\Rightarrow P(r, t - \frac{r}{c}) = \frac{\rho\ddot{V}(r_0,t)}{4\pi r}$