Seismic trace interpolation using the pyramid transform

Barry Hung, Carl Notfors and Shuki Ronen, Veritas DGC inc.*

Summary

We present a new method called the pyramid transform for seismic trace interpolation. The pyramid transform is a resampling of data in the *f-x-y* domain that gives rise to frequency dependent spatial grids with a relationship that the sampling interval is inversely proportional to the frequency. Such transformation is reversible for wave-fields. Using prediction filter, we demonstrate that seismic trace interpolation in the pyramid domain is more robust in the presence of noise and conflicting dips than interpolation in the conventional *f-x-y* domain.

Introduction

Seismic data are often spatially undersampled in 3-D exploration. Trace interpolation, a well-known solution to this sampling deficiency, is often used to estimate unrecorded traces from a spatially undersampled dataset. Various interpolation methods for handling aliased events have been proposed over the years and some of their features, such as computational efficiency and data accuracy, have been compared (Abma et al., 2003). One of the methods that was used as a benchmark in the comparison studies is called the *f-x* domain prediction filtering method, which is used routinely in data processing. This technique is based on the principle that linear events are predictable in the *f-x* domain without any prior knowledge of the dips of the events (Spitz, 1991). It operates on 2-D regular data in the temporal frequency (*f*) domain in such a way that, for an interpolation order of λ, a spatial prediction filter is designed at f with trace spacing Δx and then applied to predict data at frequency λf with trace spacing $\Delta x/\lambda$.

Based on the Spitz's method, effort has been made to enhance the computational efficiency of the method and improve the quality of the interpolation results. Porsani (1999) used the half-step prediction filters in the *f-x* domain to speed up the application process of the filters by a simple convolution operation. Wang (2002) employed 2-D prediction filters in the *f-x-y* domain for 3-D data to relax the requirement that the events must be linear for predictability. With the *f-x-y* prediction filter, events that are nonlinear in one direction but linear in another may be predicted. Hung et al. (2003) addressed the amplitude and the smearing of structural discontinuity issues by using noncausal spatial filters. While these work contribute to the improvement of the Spitz's method in one way or another, their drawback is that, in the conventional *f-x-y* method, the prediction filter needs to be estimated at each input frequency f (up to f_{max}/λ) for interpolating traces at frequency λf. Thus, the input data need to have a reasonable signal-to-noise (S/N) ratio at each frequency. Otherwise, at those frequencies with low S/N ratio, inaccurate prediction coefficients will be used for interpolating traces at higher frequencies. For field data, however, it is not uncommon to have a portion of the input frequencies that has a rather low S/N ratio. In this case, a more robust method than the conventional *f-x-y* method is needed.

In this paper, we discuss a new transform method called the pyramid transform that allows one prediction filter to be estimated from a range of frequencies, preferably the frequencies with high S/N ratio. The pyramid transform has been proposed by Sun et al. (1996) for the application of noise reduction. We extend its application to trace interpolation in this work. In the following, we first show the features of the pyramid transform and then, through the results of trace interpolation, highlight that the prediction filtering in the pyramid transform is more robust against noise and conflicting dips than in the conventional *f-x-y* method.

Theory – Pyramid Transform

To perform the pyramid transform, a 3-D dataset is first Fourier transformed in time. This transforms the data to the *f-x-y* domain. Within this domain, the data is then spatially resampled in such a way that the resulting spatial interval is inversely proportional to the frequency. That is, the lower the frequency, the smaller the number of data samples. This leads to a pyramid shaped sampling (hence the name for the transform). Moreover, the low frequencies are not over-parameterized after the transform which makes frequency dependent grids more suitable for either inversion or interpolation. The pyramid transform is based on Shannon's sampling theorem, so it is invertible. In other words, if one performs the forward and then the inverse pyramid transform, one may recover the original input data. This is illustrated in figure 1, where figure 1a shows the original data and figure 1b displays the result after the forward and inverse pyramid transform. The difference plot between the two figures is depicted in figure 1c, which shows that the pyramid transform is invertible.

Mathematically, the way to spatially resample an input data in the pyramid domain is guided by the well-known equation that relates the maximum non-aliased frequency f_m to a dip angle θ with trace interval δx:

$$f_m = \frac{v}{4\delta x \sin\theta} \qquad (1)$$

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where v is medium velocity. Rearranging equation (1) and setting $\theta=90^o$ give rise to the following equation:

$$\sin\theta = \frac{v}{4f'\delta x} = 1 \qquad (2)$$

where f' is the critical frequency. Equation (2) indicates that for frequencies below f', one may resample the spatial intervals in such a way that the new intervals Δx_f (a function of f) fulfill the relationship of $\Delta x_f = v/4f$ and yet retain dip information up to 90°; whereas, for frequencies above f', one can only obtain dip information of $\theta_{max} = \sin^{-1}(v/4f'\delta x)$ with the original trace interval δx. The use of the critical frequency f' for the pyramid transform is schematically illustrated in figure 2 where figure 2a shows the input data cube in the f-x-y domain and figure 2b the data volume after the pyramid transform. One may observe that the volume of the data has shrunk significantly after the pyramid transform. Hence, the pyramid is equivalent to the cube in the conventional f-x-y domain in terms of information amount but with smaller data volume. In addition, the base of the pyramid occurs at frequency f'; above f' the data arrangement is same as the corresponding section in the f-x-y domain.

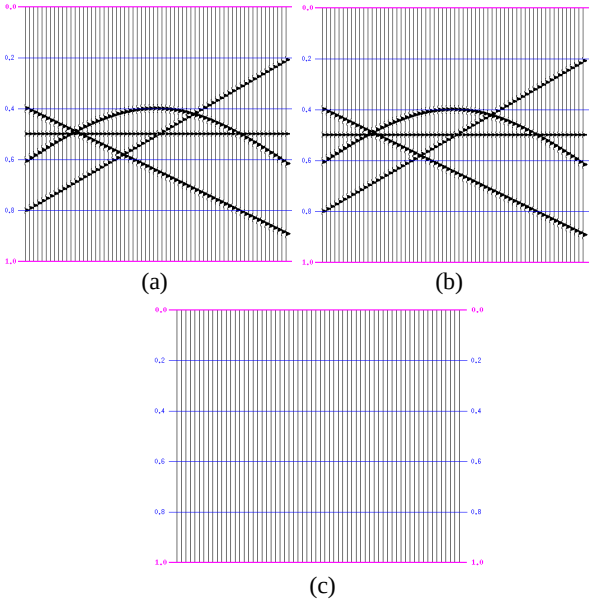


Figure 1. (a) The input data. (b) The result after the forward and inverse pyramid transform. (c) The difference plot between (a) and (b).

The feature of frequency dependent grids in the pyramid domain has an important application for spatial prediction filtering. That is, the prediction filter is independent of temporal frequency and hence a single prediction filter can be estimated from a range of frequencies below the critical frequency. This makes the estimation of the filter not only efficient, because the process needs not be repeated for each frequency, but also more stable and robust against noise, since a frequency range with high S/N ratio can always be chosen for the estimation. To illustrate this point, consider the equation of the propagator z which, for a single event, predicts the wavelet from x to $x+\Delta x$ (Sun, 1996):

$$z = e^{id2\pi f\Delta x} \qquad (3)$$

where d is the dip of the event. In the conventional f-x-y domain, z is a function of f. However, in the pyramid domain, z is frequency-invariant when Δx is set to $v/4f$ as mentioned above. For multiple events, says three events, one may use the Van Der Monde matrix equation to relate the prediction coefficients p's and the propagators z's:

$$\begin{pmatrix} z_1^2 & z_1 & 1 \\ z_2^2 & z_2 & 1 \\ z_3^2 & z_3 & 1 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} z_1^3 \\ z_2^3 \\ z_3^3 \end{pmatrix} \qquad (4)$$

Since the matrix equation is a linear system, the prediction filter obtained from the frequency-invariant z's is also independent of frequency in the pyramid domain. In practice, the estimation of the prediction filter is performed in the pyramid domain. From the filter, the frequency invariant z's are obtained. These z's are then mapped back to the original f-x-y domain to obtain the frequency dependent prediction filter pf(f) which can then be used for interpolating traces at frequency λf. Thus, the application of the prediction filter is done in the original f-x-y domain.

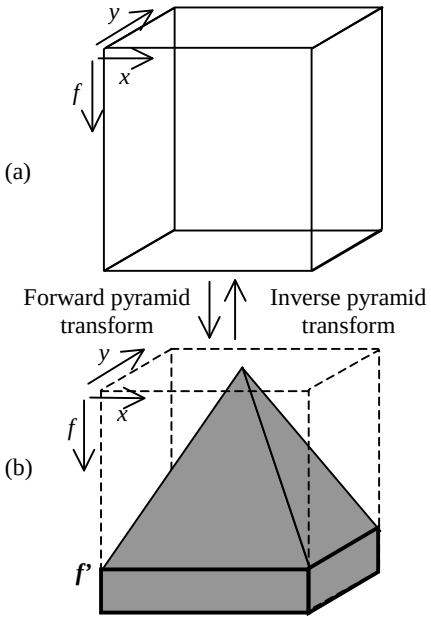


Figure 2. (a) Input data cube in the original f-x-y domain. (b) Data volume after the pyramid transform.

In the following section, we demonstrate how the pyramid transform can be applied to trace interpolation. We use only the *f-x* domain in the following examples. However, the theory which has been discussed above is applicable to both the *f-x* and *f-x-y* domains.

Examples

Figure 3a shows a linear event that is contaminated with noise (5–70 Hz) but has the highest S/N ratio in the frequency range of 10-15 Hz. The even-numbered traces of this input were decimated. After the decimation, the event is aliased above 50 Hz. The decimated traces were then reproduced by the conventional *f-x* method and the result is depicted in figure 3b. The difference between the original input and the *f-x* interpolation result is displayed in figure 3c. One may observe from the difference plot that the conventional *f-x* method does not perform well for this example even for one linear event. It is because it uses frequencies with low S/N ratio to obtain the prediction filter for interpolation, i.e., for instance, the aliased frequency of 60 Hz uses the prediction filter estimated at 30 Hz, which has low S/N ratio, for interpolation. On the contrary, when only the frequency range with the highest S/N ratio (in this case 10-15 Hz) is used for the estimation of the prediction filter in the pyramid domain, a greatly improved interpolation result, as shown in figure 3d, is obtained. From the difference plot shown in figure 3e, it is apparent that the pyramid method performs better than the conventional *f-x* method in the presence of noise.

In cases where conflicting dips as well as noise pose a problem to the conventional *f-x* method, the pyramid method may be used for generating better results. Figure 4a shows the migrated image of subline 242 of the SEG/EAGE model. Its highest S/N ratio is in the frequency range of 10-20 Hz for most parts of the area. Again, the even-numbered traces were removed and then reproduced by interpolation. Figure 4b displays the result obtained by the conventional *f-x* method. Using the frequency range of 10-20 Hz for the estimation of the prediction filter, the interpolation result obtained by the pyramid method is plotted in figure 4c. It can be seen that the pyramid method reproduces the data much better than the conventional *f-x* method. In particular, the area on top of the salt dome and the area around the salt in the timegate of 1.6-2.2 s, where conflicting dips of the events make it difficult for the conventional *f-x* method to estimate the prediction filters accurately. Moreover, the computational time needed to generate figures 4b and 4c was almost the same because, in the pyramid method, the extra time needed to transform the data to the pyramid domain is offset by the time that is saved in estimating only one prediction filter for a frequency range.

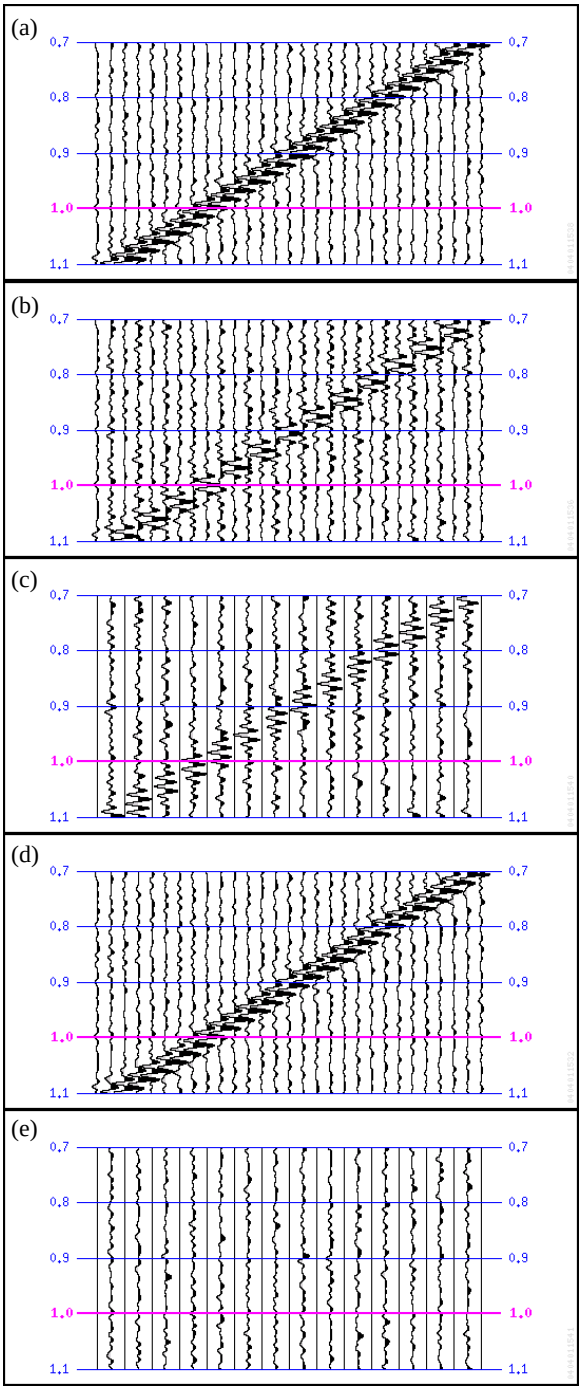
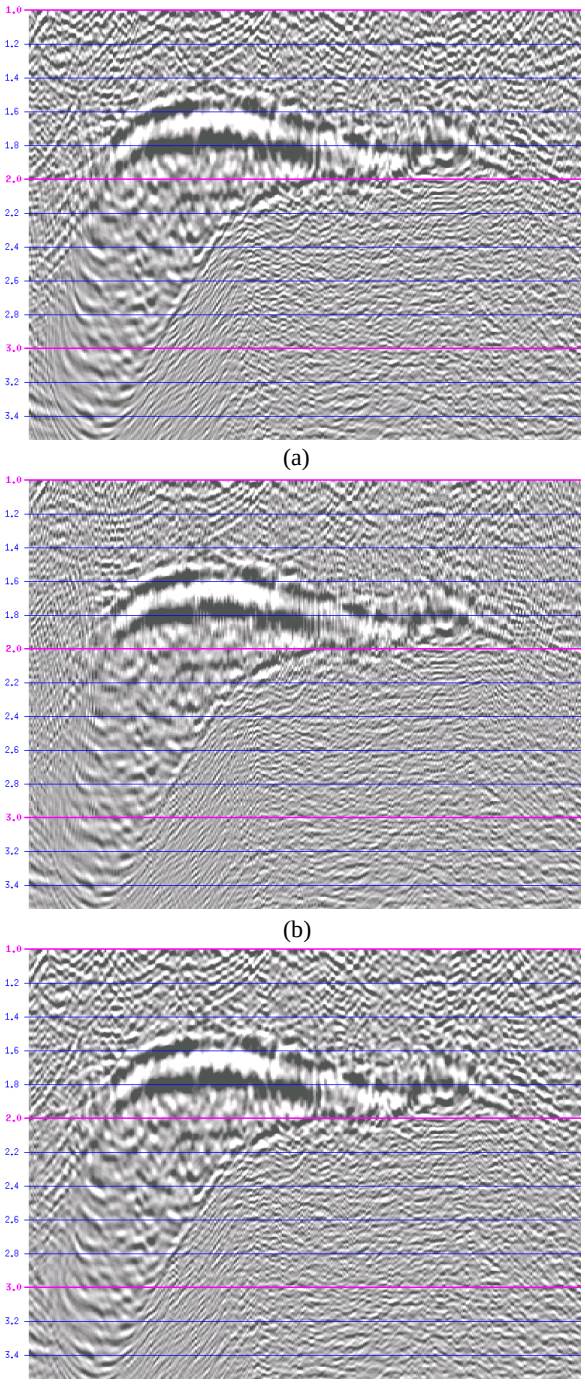


Figure 3. (a) Original input before its even-numbered traces are removed. (b) Interpolation result using the conventional f-x method. (c) The difference plot between (a) and (b). (d) Interpolation result using the pyramid method. (e) The difference plot between (a) and (d).



(a)

(b)

(c)

Figure 4. (a) Original migrated subline of the SEG/EAGE model before its even-numbered traces are removed. (b) Interpolation result using the conventional *f-x* method. (c) Interpolation result using the pyramid method.

Conclusions

We have presented a new method for seismic trace interpolation based on the pyramid transform. The pyramid transform resamples the data in the *f-x-y* domain in such a way that the new spatial interval is inversely proportional to the frequency. This gives rise to an important feature for prediction filtering that the prediction filters in the pyramid domain are frequency independent and therefore can be estimated from a range of frequencies. Using this feature, it has been shown that the pyramid method is more robust against noise and conflicting dips than the conventional *f-x-y* method for the applications of trace interpolation.

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Acknowledgments

We thank Veritas DGC for the permission to publish this work.