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Least-squares DMO and migration

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ABSTRACT

Conventional processing, such as Kirchhoff dip moveout (DMO) and prestack full migration, are based on independent imaging of subsets of the data before stacking or amplitude variation with offset (AVO) analysis. Least-squares DMO (LSDMO) and least-squares migration (LSMig) are a family of developing processing methods which are based on inversion of reverse DMO and demigration operators. LSDMO and LSMig find the earth model that best fits the data and a priori assumptions which can be imposed as constraints. Such inversions are more computer intensive, but have significant advantages compared to conventional processing when applied to irregularly sampled data.

Various conventional processes are approximations of the inversions in LSDMO and LSMig. Often, processing is equivalent to using the transpose of a matrix which LSDMO/LSMig inverts. Such transpose processing is accurate when the data sampling is adequate. In practice, costly survey design, real-time coverage quality control, in-fill acquisition, redundancy editing, and prestack interpolation, are used to create a survey geometry such that the transpose is a good approximation of the inverse.

Normalized DMO and migration are approximately equivalent to following the application of the above transpose processing by a diagonal correction. However, in most cases, the required correction is not actually diagonal. In such cases LSDMO and LSMig can produce earth models with higher resolution and higher fidelity than normalized DMO and migration.

The promise of LSMig and LSDMO is reduced acquisition cost, improved resolution, and reduced acquisition footprint. The computational cost, and more importantly turn-around time, is a major factor in the commercialization of these methods. With parallel computing, these methods are now becoming practical.

INTRODUCTION

The development of seismic reflection surveying was a tremendous achievement given the acquisition and computing technologies of the time, but compared to today's technology it had a modest start. In the beginning, 3-D surveys were a dream, and even continuous 2-D coverage along a line took some effort. The road from the early days to today's high-resolution and high-fidelity 3-D surveys is a long story marked by a few major technical milestones.

One such technology leap was multifold coverage. Cecil Green was ahead of his time in 1938 when he proposed arranging shot holes and recorder positions with common-depth-point locations (Green, 1938). It was not until much later that such coverage was practical. Mayne (1956) commercialized what he termed "multiple coverage of the subsurface," that is, overlapping measurements of the same subsurface with different shot-to-receiver offsets. How could such different measurements of the same subsurface be combined? Mayne suggested algebraic summation after applying appropriate time corrections. The computing technology of the period did not enable more than trace-sequential time correction and summation or averaging. This process, called NMO-stacking, became standard state of the art for about two decades.

As computers became faster and exploration targets became more challenging, major improvements were gradually introduced. We now know about prestack migration and dip moveout (DMO). Thus, the NMO part of NMO-stacking had a major revision. A simple thought experiment (Appendix A) shows that stacking is sometimes not the right thing to do—even if it follows DMO or prestack full migration.

Important improvements to stacking itself were also developed. Examples are median stacking and other robust averaging methods (Naess, 1979), as well as amplitude variation with offset (AVO) (Ostrander, 1984). These developments, however important, were still based on the approach that we process subsets of the unstacked data independently, produce independent images from these subsets, and later analyze the

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independent images by some averaging or by AVO analysis. In this paper, we write about an alternative approach to prestack migration/DMO and stacking. Although various names occur in the literature, in this paper we use the known terms least-squares migration (LSMig) and least-squares DMO (LSDMO).

LSMig and LSDMO have been known for over a decade but are not yet used extensively. One reason for this delay is the computational cost, but the lack of understanding of its principal idea has also been a factor. Now that it is computationally feasible to use 3-D LSDMO on a medium scale and 3-D LSMig on a small scale, it is timely to write a tutorial explaining the principle idea behind these methods. Otherwise, they might appear as too good to be true and perceived as information perpetuum mobilae which violate basic laws of sampling and aliasing (Nyquist, 1928).

One reason for this lack of understanding was a poor choice of name for LSDMO as "anti-aliasing" or "dealiasing" DMO. This caused confusion with DMO operator antialiasing, which is different from LSDMO. Operator aliasing should not be confused with data aliasing, and operator antialiasing should not be confused with interpolation and LSMig/LSDMO. In this paper, we will try to clarify the relations between antialiasing, interpolation, and LSMig/LSDMO.

This paper, which began as part of a special review volume of GEOPHYSICS, contains neither equations nor data examples. All the equations have previously been published, and sufficient examples have appeared in the literature to support the claim that the theory is practical. Instead, we focus on the conceptual understanding of LSMig/LSDMO, which is approached with the help of thought experiments and leading questions.

Leading questions

1) What is the difference between operator aliasing and data aliasing?
2) What is the difference between zero data and missing data? Why do we sometimes confuse them? Do we (should we) always treat missing data and zero data differently?
3) If we interpolate, should we do it before or after stacking?
4) Can we have reliable AVO analysis after prestack migration with irregularly sampled data?

STATE OF THE ART OF ACQUIRING AND PROCESSING 3-D DATA

From two dimensions to three: The motivation for LSMig/ LSDMO

NMO-stacking was invented when all seismic surveys were two dimensional, as were DMO and prestack migration. Two-dimensional surveys have excellent and uniform coverage in one preferred (in-line) direction but very poor spatial sampling in the other (cross-line) direction. The in-line sampling of 2-D data is often so good that even prestack data (shot profiles, receiver gathers, and common offset sections) are often well sampled. The cross-line sampling in 2-D surveys is so poor that processing completely gives up and proceeds as if the earth is flat in the cross-line direction. Processing 2-D surveys is thus simplified by the fact that unstacked data are adequately and uniformly sampled in the in-line direction. The major sampling problem in the cross-line direction is conveniently deferred to the interpreter.

Not so with 3-D surveys. 3-D data sampling is inherently irregular. In general, it is impossible in practice to have regularly and well sampled prestack 3-D data in any domain (shot, receiver, offset, or cross-spread). The difference between the in-line and the cross-line directions of 3-D surveys is less distinct. The cross-line sampling problem is no longer deferred to the interpreter, and the data processor has to wrestle with it. Moreover, data processing must address the non-uniform sampling. Marine data have irregularities due to cable feathering, and land data suffer from obstacles. During ocean-bottom surveys, it is sometimes possible to have almost regular cross-spreads or common receiver gathers, but these are never perfect and are expensive to acquire. Therefore, ocean-bottom surveys can also benefit from the ability to handle irregular sampling in processing.

Soon after 3-D seismic began, people noticed the possibility of converting some of the multifold coverage to increased output resolution and relaxing input sampling requirements. Bolondi et al. (1982) discussed how DMO could be used to reduce spatial aliasing and improve spatial resolution. They suggested that prior to DMO, zero traces could be added to common offset sections to improve and regularize spatial sampling. Although the data examples were 2-D, the motivation to this discussion was 3-D spatial sampling.

Kirchhoff summation methods

Although some people understood the potential of multifold coverage when 3-D seismic just began, these ideas were ahead of their time. In fact, to this day, LSMig/LSDMO do not have important roles in 3-D seismic data processing. Instead, when 3-D surveys started to be acquired, 2-D processing methods were used on the new 3-D data. It was found that NMO-stacking applied to 3-D data gave reasonable results. DMO and prestack migration programs, however, needed some work before they could be applied in three dimensions.

The DMO and migration programs used on 2-D data were mostly based on finite-difference (FD) and Fourier transform (FT) methods. Such FD/FT methods require sorting the data to a preferred domain and uniform sampling in that domain. Typical examples are prestack migration of shot profiles, and DMO of common offset sections. Kirchhoff migration methods (Hagedoorn, 1954) preceded FD/FT methods (Claerbout, 1970). Early DMO methods were FD (Yilmaz and Claerbout, 1980; Judson et al., 1995), but Kirchhoff DMO followed quickly (Deregowski and Rocca, 1981) before FT DMO (Hale, 1984) was established as the standard method in two dimensions. However, all migration and DMO Kirchhoff methods had marginal usage in two dimensions. They could not compete with more efficient and more accurate FD/FT programs. The advantages of Kirchhoff methods, that no sorting was required and that irregular sampling was apparently tolerated, were not very important in two dimensions. It was only later, when 3-D surveys started to produce large quantities of irregularly sampled data, that Kirchhoff methods had their finest hour.

Considerable effort was required to give Kirchhoff summation programs the same fidelity that FD/FT programs had. There were three major issues that had to be addressed: (1) the

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kinematics of the operator, (2) the amplitude of the operator, and (3) the sampling (or antialiasing) of the operator.

In this paper, we focus on sampling and aliasing, so emphasis will be put on the third issue. Recommended reading is Geoltrain and Brac (1993) for kinematic analysis; and Sorin and Ronen (1989), Bagaini (1993), Black et al. (1993), Tygel et al. (1998), and Bleistein et al. (1999) for amplitude analysis.

The main issue of interest in this paper is the operator sampling which had to be addressed. This issue was known from 2-D Kirchhoff methods, but the motivation to seriously address it was not there until Kirchhoff methods were really needed to process 3-D data. FD/FT programs produce no artifacts when applied to adequately sampled data. Early implementations of Kirchhoff summation methods produced artifacts, usually a precursor noise. Hale (1991) provided the vertical component of DMO operator antialiasing. In essence, high frequencies had to be removed from the steeply dipping parts of the operator. It was noted that FT DMO had such antialiasing built-in with no compromise of resolution. Similar solutions were provided for prestack migration and summarized by Gray (1997).

That was not the end of the story of DMO operator antialiasing. There is also a horizontal component of antialiasing. The DMO operator is very narrow in map view. In a constant velocity medium, it is nonzero only on the shot-receiver line. Such a narrow line could be spatially aliased much like line rasterization is often aliased—a known problem in image processing. The solution to line rasterization aliasing is to allow the line to be wider than the raster size using gray tones. Similarly, the DMO operator was made wider than the bin size. In other words, bins which are not on the shot-receiver line, but close enough, receive a weighted contribution. Wide DMO benefits on real data examples were presented by Beasley and Mobley (1998) and Huard and Spitz (1998). With proper operator antialiasing, Kirchhoff summation programs have been perfected to the point that applied to well-sampled data, their results were equivalent to benchmark FD/FT results. Of course, the point was that the Kirchhoff programs could be applied to irregularly sampled unsorted data. And indeed they were.

It soon became apparent that even properly implemented programs produce significant artifacts when applied to irregularly sampled data. These artifacts could no longer be blamed on operator aliasing, and often resembled the geometry or coverage maps. These effects became known as "acquisition footprint."

In-fill acquisition, redundancy editing, flex binning, and interpolation

Acquisition footprint can be avoided if the coverage is uniform. Great efforts, at great cost, are made to design and operate surveys such that the coverage is uniform. Offshore, where cable feathering cannot be fully controlled, the coverage is monitored during the survey. Gaps or areas of low coverage occur when feathering between adjacent boat passes does not match. Areas of high coverage happen as often as areas of low coverage. To fill gaps, additional boat passes are done at significant cost. The data thus obtained are known as "in-fill data." Often in-fill acquisition is compromised to save cost. Also, some data are found to be noisy or have faulty positioning, so they have to be removed from the survey. Such data are usually reshot, but sometimes the problem is discovered after leaving the area and a gap may remain. Typical surveys, therefore, have some areas of low fold, even some gaps, and many areas of high fold.

Both high-fold and low-fold areas can cause acquisition footprint. The conventional solution for high-fold areas is to reduce the amount of such data so that coverage would be uniform. The simplest way is to reject redundant data, a process known as "redundancy editing." If the redundant data share the same offset and azimuth, more sophisticated methods can be applied such as partial stacking before prestack migration or DMO. Usually, such redundant traces do not share offset and azimuth and are (arbitrarily) removed from the data. This is not elegant, but it is very effective to reduce acquisition footprint.

Low-fold areas and gaps are a more serious problem. These are very common when data are processed in offset or angle bands for AVO. While these artifacts sometimes "stack out," often they don't. Fidelity and AVO analysis can be severely compromised. Conventional solutions for low-fold areas are flex binning and interpolation.

Flex binning is simply copying traces from adjacent bins to make up for a small number of traces in a problematic bin. Flex binning can be regarded as nearest neighbor interpolation. More reasonable interpolation may be linear interpolation, where the interpolated data are a weighted average of neighboring data. Yet more sophisticated interpolation methods are based on prediction filters, which are estimated from neighboring data (Spitz, 1991). These methods, however, often have difficulties with irregular sampling. Special methods to interpolate irregularly sampled data have been proposed with some success (Kinneging et al., 1991; Jakubovicz, 1994). However, a problem with interpolation of prestack data is that they are not sampled from a single experiment. Instead they are samples from wavefields of many experiments with a single earth model, a fact which is not recognized by interpolation methods. Often, better results are obtained by imaging the data with larger bins and interpolating poststack.

Biondi et al. (1998) proposed a related method for prestack sampling regularization called "azimuth move out" (AMO). AMO is a member of a family of prestack continuation operators. Other examples are offset continuation (Bolondi et al., 1982) and shot continuation (Bagaini and Spagnolini, 1996). AMO rotates the azimuth and modifies the offset producing offset cubes instead of the zero-offset cubes produced by DMO. It can be used to prepare data for prestack migration using a 3-D FD/FT method. It is important to realize that, like DMO and prestack migration, AMO works well when its input is adequately sampled. So, by itself, AMO cannot be considered a solution to irregular data sampling.

Taken together, in-fill acquisition, redundancy editing, flex binning, and interpolation are effective in preparing data for prestack migration or DMO-stack with minimal artifacts. This combination provides a high-quality but high-cost solution, with cost coming mainly from in-fill acquisition.

Normalized migration/DMO

Processors noticed that NMO-stacking often suffered less than DMO and migration from acquisition footprint problems. In particular, redundancy editing was not needed for NMO-stacking but was essential for DMO-stacking. The reason was that stacking programs have optional (usually default)

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normalization, typically division by the fold. In other words, stacking has always been averaging and not only summing. Such normalization was simple to calculate and implement for NMO-stacking. For DMO and prestack migration, however, it is more tricky to normalize because the contributions to every output bin were not equal and could not be simply counted. Beasley and Klotz (1992) proposed the equalized DMO method in which normalization was calculated based on multidimensional binning in the time-midpoint-offset-dip-azimuth domain. Equalized DMO produced a significant improvement when applied to real data. Various competing normalization methods were then developed for both DMO and prestack migration and gave similar results. AMO can also benefit from normalization (Chemingui and Biondi, 1997). An interesting variation was proposed by Navarro et al. (1999) in which the normalization was applied to the data before prestack migration.

Normalized DMO/migration greatly reduced the need for redundancy editing. However, the performance of these corrections in areas of gaps and low fold falls below that of LSMig and LSDMO.

LEAST-SQUARES MIGRATION AND LEAST-SQUARES DMO

Processing, modeling, and inversion

Processing programs, like migration and DMO, produce an earth reflectivity image from input data. They are based on wavefield extrapolation operators. DMO is based on offset extrapolation inwards towards zero offset; migration is based on downward continuation, or reverse time extrapolation. A major difficulty of 3-D processing is the application of wavefield extrapolation operators to irregularly sampled data. Such extrapolators work well on regularly and adequately sampled data, but fail when applied to inadequately sampled data.

Modeling programs estimate synthetic data from input earth models. One example is reverse migration (or demigration), which calculates synthetic data from a given earth reflectivity model. Another example is reverse DMO, which extrapolates data outward in the offset direction from zero offset towards the far offsets. Modeling programs do not suffer from irregular sampling problems or aliasing of their input models. The input earth model for reverse DMO is a zero-offset image—the theoretical data that would be recorded had one been able to conduct a seismic survey with zero-offset between shot and receiver. Clearly this theoretical zero-offset seismic survey would be well sampled (for example, the shot-receiver pair would be placed on a regular grid at the center of every bin). The bin size would be small enough so that reflectors with the highest possible dip would be adequately sampled up to the highest frequency in the data. Similarly, the input reflectivity model for a demigration program would be given with adequate sampling.

Modeling, therefore, is a better posed problem than processing. But is modeling important? After all, we usually want to process field data and not to generate synthetic data.

As we can see in the thought experiments, demigration can be used to check whether a proposed image fits the data. More generally, the demigration operator can be expressed as a matrix and can be inverted. Iterative inversion, such as conjugate gradient algorithms, would repeatedly use the difference between estimated data from the current image and real data to update the image until convergence. LSMig and LSDMO are such inversion methods. The implied system of equations is usually overdetermined and contains inconsistent noise, so the fit can be accomplished by minimizing the difference between estimated data and real data.

LSDMO was first proposed by Ronen (1985), the utility of the log stretch transform was better understood later (Ronen and Liner, 1987), and the full theory was published by Ronen et al. (1991). Constraints were added to the inversion when the method was commercialized (Ronen et al., 1995) with encouraging results (Brink and Ronen, 1998) of reduced acquisition footprint and increased resolution.

Early suggestions towards LSMig were made by Miller et al. (1987), LeBras and Clayton (1988), and Beydoun and Mendes (1989). Recently, practical implementations and encouraging results were published by Duquet et al. (2000) and by Nemeth et al. (1999).

LSMig and LSDMO are not velocity analysis techniques. Once the velocity is known, LSMig will provide a high-fidelity migrated image taking the acquisition geometry into account. LSDMO will provide a high-fidelity zero offset image, also taking the acquisition geometry into account. Both LSMig and LSDMO depend on accurate velocity analysis. If the velocity model is poor, LSMig and LSDMO will fail (even if the acquisition footprint is not challenging). Velocity analysis can be seen as another inversion process in which a velocity model that best fits the data is found. However, LSMig and LSDMO must not be confused with velocity inversion, velocity analysis, velocity model building, or tomography. LSDMO and LSMig do not contain velocity analysis, they depend on it to have been done well.

LSDMO or LSMIG?

LSMDO or LSMig is a similar question to "DMO or prestack full migration?"

Some advantages of LSDMO are that it is less sensitive to approximations in the velocity model, less compute intensive, and easier to constrain.

On the other hand, advantages of LSMig include greater accuracy in the presence of strong velocity variations such as with sub salt targets. And it is applicable to data recorded on deep ocean bottoms and in boreholes.

For a particular data set, which is more appropriate, LSDMO or LSMig? This largely depends on structural complexity and lateral velocity variations. If these are severe enough to require prestack depth migration, then LSDMO will probably fail and LSMig is the right choice (assuming a depth migration algorithm is built in). For common cases of mild structure and lateral velocity variation, LSDMO is appropriate and less expensive than LSMig.

Normalized migration/DMO and least-squares migration/DMO

It can be shown that migration/DMO without normalization is equivalent to application of the transpose of the matrices that LSMig/LSDMO invert. Similarly, normalized DMO/migration is approximately equivalent to using a diagonal approximation to the inverse, or alternatively to column scaling preconditioning. The diagonal approximation can be applied trace sequentially poststack, neglecting the off-diagonal terms which cannot be applied trace sequentially.

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L1Mig and L1DMO

Least-absolute-deviation-fit (L1) solvers are more robust than least-squares (L2) solvers. Therefore L1Mig/L1DMO would be more robust than LSMig/LSDMO. This can be important in the presence of swell noise and ground roll. These types of noise have mostly low frequencies, which are well constrained so we can afford to ignore many equations that are suspected of describing noisy data. Practical L1 solvers, such as iteratively reweighted least squares, have a cost which is a few times the cost of L2 solvers. L1DMO is possible today, but L1Mig may be a few years ahead of today's computing technology.

ANSWERS TO THE LEADING QUESTIONS

Data aliasing and operator aliasing

Operator aliasing is what causes precursor artifacts on Kirchhoff DMO and migration when regularly sampled data are input. Having modified the Kirchhoff program so that no artifacts are produced for adequately sampled input, spatial aliasing artifacts which are produced from irregularly sampled data are due to data aliasing.

Missing data and zero data

Of course missing data are not zero data. The fact that we do not have a station somewhere is not the same as the fact that if we had a station in that place we would record zero data. However, when we process data by NMO/DMO/migration and stacking, there is no difference between missing data and zero data—that is until we get to normalization, when we want to divide by the number of live data and not count missing or muted data that went in as zeros. Like normalized DMO/migration, LSDMO/LSMig also make a distinction between missing data and zero data. Combined with wide-angle reflection muting, this often causes confusion.

The role of interpolation

Flex binning and interpolation have important roles in today's state of the art processing. With the introduction of LSDMO/LSMig, we expect to see this role diminished. However, interpolation will remain important for suppression of coherent noise such as multiples and ground roll. LSDMO and LSMig can contain interpolation in the form of predictability constraints. A reasonable a priori assumption is that the earth model is self predictable. Spitz (1991) developed a clever interpolation method based on prediction filters estimated at low frequency with a certain sampling interval, and applied in double the frequency with half the sampling interval. A generalization of this method can be based on frequency dependent grids with a sampling interval inversely proportional to the frequency. With such frequency dependent grids, the prediction filters are the same for all frequencies. This can be used to constrain the earth model in LSMig/LSDMO. Such predictability constraints can be called "interpolation during imaging."

What about AVO?

We want to perform prestack migration before AVO. A practical method is to partition the data to offset bands and to migrate each band separately. One problem, and a possible cause for many failures, is that the coverage within narrow offset bands or angle bands is irregular, and acquisition footprint artifacts may overwhelm true AVO variations. A possible solution is to use LSMig and to extend the earth model to include amplitude versus angle variations and invert a more elaborate demigration which generates synthetic data with AVO. This inversion can become impractical if the demigration operator is not nonlinear, but linear approximations seem sufficiently accurate for most applications. A more practical possibility is to use LSMig in offset bands. In such cases, LSMig can be used to improve the imaging of each offset band. Alternatively, LSDMO can be used on each offset range. If each offset band is migrated after LSDMO and before AVO, this is approximately equivalent to LSMig before AVO.

How much data do we need?

The case-dependent answer to this question has a significant economic impact. Thought experiment 2 (Appendix B) demonstrates that the amount of data we need for imaging can be surprisingly small. However, the fact is that coherent noise usually requires more sampling than the reflections. For example, ground roll and swell noise require more sampling than what is needed for the reflection events. Another important example is multiple suppression. More sampling in the offset direction is required to stack-out the multiples than to stack-in the primary reflections.

If we acquire large quantities of data in order to suppress coherent and random noise, would there still be a need for methods like LSMig/LSDMO whose strength is in requiring minimal data for the imaging of the signal? It is true that LSMig and LSDMO promise higher returns with noise-free data, but they still have a potential to save significant acquisition cost. Fine sampling of ground roll can be obtained by small shot and receiver intervals in cross-spread geometry. The ground roll can then be removed by 3-D dip (FKK) filtering. Combining multicross-spread data can be problematic due to variable offset and azimuth distribution; this can degrade AVO. LSMig/LSDMO can be useful at this stage. Similarly, towed streamer marine data in-fill cost can be reduced using LSMig/LSDMO. The offset sampling needed for multiple attenuation will typically not require in-fill data. Moreover, often the most effective way to remove coherent noise is by removing the contaminated traces from the data set. For example, ground-roll noise often contaminate only the near offsets. If the groundroll cannot be filtered (because it is spatially aliased), it is possible to remove it by removing the near offset traces, noise and signal together, from the data. This will create a coverage problem, but LSDMO or LSMig can still produce a good image from the remaining data. Similarly, multiples can be suppressed by removing the near offsets. Swell noise can be removed by rejecting low frequencies of contaminated traces. Coverage problems that are created by the removal of signal that coincides with the noise can be solved by LSDMO/LSMig.

CONCLUSIONS

LSMig and LSDMO are inversion methods which build earth models that best fit all the available data. Compared to conventional state of the art processing of migration or DMO

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and stack, LSMig and LSDMO promise reduced acquisition effort, reduced acquisition footprint, and increased resolution. Current data processing practice is an approximation to the least-squares methods, and the accuracy of the approximation depends on the acquisition geometry. Standard practices of survey design, in-fill acquisition, redundancy editing, and interpolation have evolved to provide the geometry required by this approximation. Realization of the economic potential of the least-squares methods requires, therefore, not only fast and cheap computing but also the understanding that these methods can significantly reduce acquisition effort.

Arguably, LSDMO and LSMig are among the most significant data processing developments since CDP stacking and AVO analysis. The basic idea behind the least-squares methods have been known for over a decade; it is now becoming possible to use them in practice.

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APPENDIX A

THOUGHT EXPERIMENT 1

Consider a 2-D line composed of two common offset (COS) panels. One COS is full of live data, the other contains a single live trace. Prestack migration of the first COS gives a reasonable image, but migration of the second COS gives a family of migration smiles. Stacking smears the two and degrades the one good image. The migration smiles on the final image are obvious artifacts. Since the first COS had sufficient data, it seems we would be better off without the second COS. Ignoring that trace we would produce a more reasonable image from the well-sampled COS. So we ignore the additional trace (redundancy editing) and migrate the remaining well-sampled COS panel.

Now, suppose that we have reversed the common offset migration program to create a common offset demigration program. If we apply this demigration program to the image, we could regenerate the COS panel that we migrated. By definition (and ignoring some minor invertability issues), we will reproduce the data we started from. The output offset is just a parameter of the common offset demigration program,

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so we can apply demigration to produce any desired COS. In particular, we can regenerate the COS containing the single trace we have ignored as redundant. Even if that additional trace had not yet been used in the processing, we could verify that the image is consistent with this trace by comparing the data trace to the one produced by demigration. If there is no AVO variation, and if the well-sampled COS has no noise (perfect COS, perfect demigration), we would simply confirm that the additional trace is redundant.

Now, assume that instead of having a well-sampled COS and an additional trace we have two COS panels, each missing 25% of its traces in an irregular, random way. We know that each common offset panel contains some redundant and some unique information about the true earth image. Neither of the panels is a complete data set that we should migrate independently. If we ignore the problem and use prestack migration and stack, we would produce a mix of real reflections and artifacts, probably a poor result. A standard fix would be to interpolate the two COS panels, then apply migration. The interpolator could be very smart, like some of the methods in Claerbout (1992). But any interpolator that does not have some migration or offset continuation built in may go astray when applied independently to the two COS panels. A really smart interpolator would need to understand that both COS panels exist and that they contain information about a common image. Such an interpolator could fill in the COS panels, then output regularly sampled data that we can process easily. But since this smart interpolator must know about the common image and would have migration built in, it might just as well put out the best-fit image consistent with both COS panels. This smart interpolator exists and has a name: least-squares migration.

An iterative implementation of LSMig can start from an initial estimate of the earth image obtained, for example, by conventional migration. Then it can use a demigration subroutine to produce data that would be recorded if this earth image were true. If the estimated data fits the recorded data, then we are done, the earth image is correct. However, if there is a difference between the real data, and the estimated data, then the earth image can be iteratively updated to improve the fit to the data.

The earth image thus built does not have to be migrated. If we build a zero-offset image, the process is called LSDMO. The zero-offset image generated by LSDMO would require migration.

APPENDIX B

THOUGHT EXPERIMENT 2

The objective here is to show that we can, sometimes, image the earth's interior with very little data—if the data are used in an optimal way. This thought experiment was inspired by Neidell (1994).

We assume a simple but unknown earth reflectivity model, constant velocity, and no noise. The simple earth reflectivity model is a single point diffractor. If it is known a priori that there is just one point diffractor, then two prestack traces in two dimensions and three in three dimensions suffice to calculate the location. But what if we do not know that it is just a single point diffractor?

If we acquire four traces, the data set can be described by Figure B-1.

If we migrate these data, each trace will contribute a migration ellipse, and all the ellipses will intersect on the point diffractor (Figure B-2).

Migration will produce the four ellipses which indeed reinforce each other on the true point but also produce significant artifacts. The artifacts, in the form of migration ellipses, are due to the sparse acquisition geometry. But aren't they also due to the way we processed the data?

An experienced interpreter may look at Figure B-2 and realize that there might be just a single point diffractor as shown in Figure B-3.

Will this experienced interpreter be able to check the conjecture? In this simple case it could be demonstrated with a demigration program. There are two candidate models. One is shown in Figure B-2 and the other is shown in Figure B-3. The interpreter could apply the demigration program to each candidate image and measure how well the synthetic data fit the data given in Figure B-1.

Applied to the Figure B-3 image (just the point diffractor), the demigration program would generate a perfect fit to the data. Applied to the Figure B-2 image, the demigration program would generate a less than perfect fit to the data because

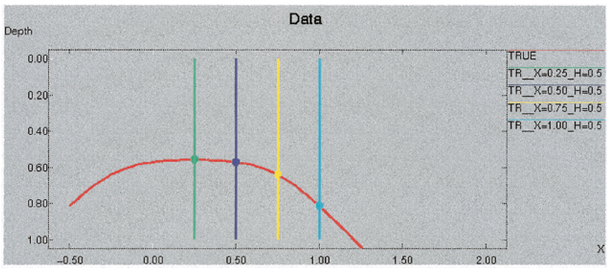

FIG. B-1. Data for a single point diffractor. The geometry is assumed to be four traces with a common offset. Each trace contains a single event. The sampling of the full diffraction curve (in red) is poor.

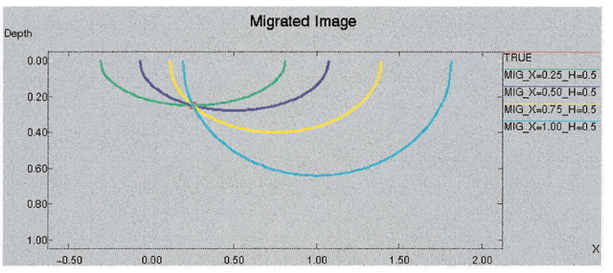

FIG. B-2. Migrated image of the data in Figure B-1. Each trace contributes a migration ellipse. The same color coding is as in Figure B-1. All the migration ellipses intersect at the true diffraction point in red. A large number of such migration ellipses will reinforce (stack-in) at the diffractor and cancel (stack out) everywhere else. It seems, however, that with only four traces, the image is overwhelmed by artifacts.

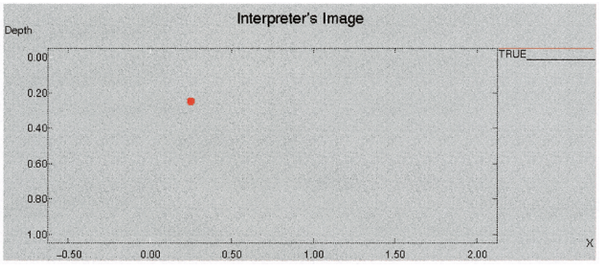

FIG. B-3. An interpreter's image of the earth model. The conjecture is that given the sparse geometry and the migration results of Figure B-2, there is really only a single point diffractor in the earth.

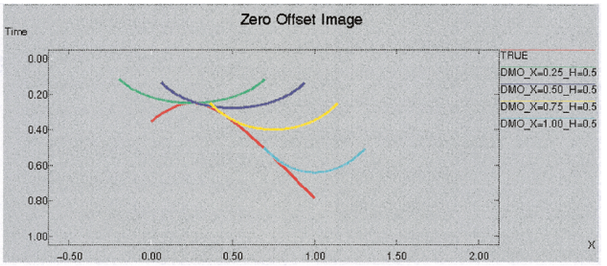

FIG. B-4. Zero-offset image of the same data and earth model of Figures B-1 and B-2. Each trace contributes a DMO ellipse. A large number of such DMO ellipses will cancel everywhere except on the diffraction hyperbola. But it seems that with just four traces, the image is overwhelmed by artifacts.

in addition to the true events, there will be weak reflections from the other ellipses. It will thus be possible to prove that earth model B-3 is better than earth model B-2—not only is it simpler, it better fits the data.

A similar thought experiment can be done in the zero-offset domain. The DMO-stack result (Figure B-4) contains DMO ellipses (instead of migration ellipses), and the true image is a single hyperbola (instead of a single point). If we build an image in the zero-offset domain, we would use a reverse DMO (instead of a demigration) program to generate synthetic non-zero offset data that we would compare to the given data. Having built a zero-offset image containing a single hyperbola, we would than migrate it to find the single point diffractor.

It can be argued that a single point diffractor is a special case and that if there was a more complicated earth reflectivity, then four data traces would not do much good to image real data. Yet, they might do some good in a noise-free thought experiment like this one to demonstrate that when the geometry of the data is irregular, stacking is suboptimal processing, even after perfect prestack migration with exact velocity.

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